INSTRUCTIONS

1. Read all instructions carefully before you start writing.
2. This paper consists of four (4) questions.
3. Answer all questions.
4. Total marks for each question are indicated at the beginning of each question.
5. This question paper is applicable to both full-time, part-time and distance students.

This paper consists of 2 pages, including this front page.
Question 1 [25 Marks]

Given $\alpha$ is non-income tax, $\beta$ is income tax, $\delta$ is marginal propensity to consume, $\gamma$ is autonomous consumption, $Y$ (national income), $I_0$ (investment) and $G_0$ (government expenditure)

1. Formulate the equations needed to find the reduced form of equilibrium income ($Y_e$). (5)
2. Do comparative static analysis to find the effect of income tax, non-income tax and government spending on equilibrium income. (15)
3. If $\beta = 0.2$; $\alpha = 20$; $\gamma = 80$; $\delta = 0.25$; $I_0 = 45$; $G_0 = 50$. Find the effects of lump sum tax increase by $1$ billion? (5)

Question 2 [20 Marks]

In a three-industry economy, it is known that industry I uses 30 cents of its own product, 40 cents of commodity III and 20 cents of commodity II to produce a dollar’s worth of commodity I. Industry II uses 15 cents of its own product, 35 cents of commodity III and 45 cents of commodity I to produce a dollar’s worth of commodity II. While industry III uses none of its own product and commodity I, but uses 25 cents of commodity II in producing a dollar’s worth of commodity III. The open sector demands NS 1,000 billion of commodity I, NS 800 billion of commodity II and 1200 billion of commodity III.

a) Write out the input matrix, and the specific systems of equations for this economy. (5)
b) Find the solution output levels by Cramer’s rule. (15)

Question 3 [20 Marks]

i. Use Jacobian determinants to test the existence of functional dependence between the paired functions.
   
   \[ y_1 = 3x_1^2 + x_2 \]
   \[ y_2 = 9x_1^3 + 6x_1^2(x_2 + 4) + x_2(x_2 + 8) + 12 \]  
   (5)

ii. Optimise the following function. a) find the critical value for the first order condition and b) the high-order Hessian:
   
   \[ y = 4x_1^2 - 7x_1 - x_1x_2 + 8x_2^2 - 5x_2 + 2x_2x_3 + 4x_3^2 + 2x_3 - 4x_1x_3 \]  
   (15)

Question 4 [35 Marks]

1. Determine the optimal solution to the linear programming problem:
   
   (a) Minimize $Z = 3x + 6y$ subject to $4x + y \geq 20; x + y \leq 20; x + y \geq 10$ and $x; y \geq 0$. (5)
   (b) Minimize $\pi = 3F + 3.6S$ subject to $30F + 50S \leq 450; F + S \leq 10$ and $F; S \geq 0$. (5)

2. Determine the following integral, using the substitution method.
   
   Given $\int 10 x(x^2 + 3)dx$. (5)

3. Given the marginal propensity to import $M'(Y) = 0.1$ and information that $M = 50$ when $Y = 0$, find the import function $M(Y)$. (5)

4. Given the marginal propensity to consume $C'(Y) = 8 + 0.1Y^{1/2}$ and information that $C = Y$ when $Y = 100$, find the consumption function $C(Y)$. (5)

5. Evaluate $\int_0^4 \left( \frac{1}{3+3x} + 2x \right) dx$ where $x \neq -1$ (10)