| QUALIFICATION : **BACHELOR OF SCIENCE IN APPLIED MATHEMATICS AND STATISTICS** | |
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| QUALIFICATION CODE: **07BSAM** | LEVEL: **7** |
| COURSE: **NUMERICAL METHODS 2** | COURSE CODE: **NUM702S** |
| DATE: **NOVEMBER 2024** | SESSION: **1** |
| DURATION: **3 HOURS** | MARKS: **100** |

### FIRST OPPORTUNITY: QUESTION PAPER

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| **EXAMINER:** | *Dr SN NEOSSI-NGUETCHUE* |
| **MODERATOR:** | *Prof S.S. MOTSA* |

### INSTRUCTIONS:

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations. All numerical results must be given using 5 decimals where necessary unless mentioned otherwise.
3. All written work must be done in blue or black ink and sketches must be done in pencil

### PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

### ATTACHEMENTS

None

**This paper consists of 3 pages including this front page**

**Problem 1** [19 Marks]

**1-1.** Find the Padé approximation $R_{2,2}(x)$ for $f(x) = \ln(1+x)/x$ starting with the MacLaurin expansion

$$f(x) = 1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \frac{x^4}{5} - \cdots .$$  [12]

**1-2.** Use the result in **1-1.** to establish $\ln(1+x) \approx R_{3,2} = \dfrac{30x + 21x^2 + x^3}{30 + 36x + 9x^2}$ and express $R_{3,2}$ in continued fraction form.  [7]


**Problem 2** [30 Marks]

For any non negative interger $n$ we define Chebyshev polynomial of the first kind as

$$T_n(x) = \cos(n\theta), \quad \text{where } \theta = \arccos(x), \text{ for } x \in [-1, 1].$$

**2-1.** Show the following property:  [5]

$$T_n \text{ has } n \text{ distinct zeros } x_k \in [-1, 1] : x_k = \cos\left(\frac{(2k+1)\pi}{2n}\right) \text{ for } 0 \leq k \leq n-1.$$

**2-2.** Compute the expressions of the first five Chebyshev polynomials of the first kind $T_0, T_1, T_2, T_3$ and $T_4$.  [3]

**2-3.** Given the trucated power series $f(x) = 1 + 2x - x^3 + 3x^4$.
   (i) Economise the power series $f(x)$.  [3]
   (ii) Find the Chebyshev series for $f(x)$.  [5]

**2-4.**  (i) Show that the following function $f$ is even and use an appropriate result to find its Fourier series

$$f(x) = \begin{cases} \dfrac{\pi}{2} + x, & \text{for } -\pi \leq x < 0, \\ \dfrac{\pi}{2} - x, & \text{for } 0 \leq x < \pi. \end{cases}$$  [12]

   (ii) Set $x = 0$ and conclude that $\dfrac{\pi^2}{8} = 1 + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \dfrac{1}{7^2} + \cdots .$  [2]


**Problem 3** [27 Marks]

**3-1.** Given the integral

$$\int_0^3 \frac{\sin(2x)}{1 + x^5} dx = 0.6717578646 \cdots$$

**3-1-1.** Compute $T(J) = R(J, 0)$ for $J = 0, 1, 2, 3$ using the sequential trapezoidal rule.  [10]

**3-1-2.** Use the results in **3-1-1.** and Romberg's rule to compute the values for the sequential Simpson rule $\{R(J, 1)\}$, sequential Boole rule $\{R(J, 2)\}$ and the third impprovement $\{R(J, 3)\}$. Display your results in a tabular form.  [12]

**3-2.** State the three-point Gaussian Rule for a continuous function $f$ on the interval $[-1, 1]$ and show that the rule is exact for $f(x) = 5x^4$.  [5]

**Problem 4** [24 Marks]

**4-1.** The matrix $A$ and its inverse are $A^{-1}$ are given below

$$A = \begin{bmatrix} 1/2 & -1 \\ -1 & 1 \end{bmatrix}, \qquad A^{-1} = \begin{bmatrix} -2 & -2 \\ -2 & -1 \end{bmatrix}.$$

• Use the power method to find the eigenvalue of the matrix $A$ with the smallest absolute value. Start with the vector $\mathbf{x}^{(0)} = (1,0)^T$ and perform two iterations. [6]

**4-2.** Use Jacobi's method to find the eigenpairs of the matrix

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

[18]

God bless you !!!