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QUALIFICATION: BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS						
QUALIFICATION CODE: 07BSAM	LEVEL: 7					
COURSE: MATHEMATICAL MODELLING 2	COURSE CODE: MMO702S					
DATE: NOVEMBER 2024	SESSION: 1					
DURATION: 3 HOURS	MARKS: 150 (To be converted to 100%)					

# FIRST OPPORTUNITY: EXAMINATION QUESTION PAPER

**EXAMINER:** 

Prof Sunday A. Reju

MODERATOR:

Prof Oluwole D. Makinde

# INSTRUCTIONS

- 1. Answer all questions on the separate answer sheet.
- 2. Please write neatly and legibly.
- 3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
- 4. No books, notes and other additional aids are allowed.
- 5. Mark all answers clearly with their respective question numbers.
- 6. Use of COMMA is NOT ALLOWED for a DECIMAL POINT.

#### PERMISSIBLE MATERIALS

1. Non-Programmable Calculator

# **ATTACHMENTS**

NONE

This paper consists of 3 pages including this front page.

## QUESTION 1 [30 MARKS]

- (a) Comprehensively discuss Post-optimality analysis in linear optimisation modelling and the basic question it answers. (3 Marks)
- (b) A college hostel furniture maker realises a net unit profit of \$25 per desk and \$30 per locker. Assume that he has up to 680 metres of wood to devote weekly to the project and up to 125 hours of labour. He estimates that it requires 20 metres of wood and 5 hours of labour to complete a desk and 30 metres of wood and 3 hours of labour for a locker. Moreover, he has signed contracts to deliver two desks and one locker every week. Formulate and solve the model maximizing his profits for desks and lockers.

(7 Marks)

(c) Define post-optimality analysis for the problem defined in (b) and hence discuss the analyses for changes in his profits on both lockers and desks stating your observations for the two furniture items and providing a summary for your two analyses.

(10.5 Marks)

(d) State the mathematical definition of RELIABILITY and its expression. Specifically describe Serial and Parallel systems and their reliability equations. Hence for a system whose component reliabilities are 0.94 and 0.97, determine the system reliability when the components are assembled serially and in parallel, stating your observations for the two assemblies. (9.5 Marks)

## QUESTION 2 [60 MARKS]

- (a) Discuss the basic characteristics of Queuing system and state three basic performance measures of the system. (10 Marks)
- (b) Consider a single server freight system model where ten vans arrive at a warehouse to unload cargo according to the following time data (in minutes):

VANS#	VAN 1	VAN 2	VAN 3	VAN 4	VAN 5	VAN 6	VAN 7	VAN 8	VAN 9	VAN 10
Inter-Arrival	20	55	64	184	210	40	35	10	90	50
Times										
Unloading	50	45	62.5	75	85	70	90	30	50	60
Duration										

By constructing an appropriate simulation table, using the table headers below, obtain the following performance measures of the warehouse unloading service system (correct to 2 decimal places): (50 Marks)

Van #	IAT	ARRT	UD	SST	QLA	WT	TTW	SIT	DEPART
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where the header acronyms are respectively, Inter-Arrival Time, Arrival Time, Unloading Duration, Service Start Time, Queue Length on Arrival, Wait Time, Total Time at Warehouse, Server Idle Time.

- (i) Average wait time (WT).
- (ii) Average unloading duration (UD).
- (iii) Average time spent at the warehouse (TTW).
- (iv) Percentage of time the server is idle (SIT)
- (v) When do the 3<sup>rd</sup>, 6<sup>th</sup> and 9<sup>th</sup> vans leave the warehouse?

## QUESTION 3 [20 MARKS]

- (a) Discuss Markov Chain and its associated concepts, providing specifically a general 2-state Markov process representation. (7 Marks)
- (b) Consider a model for the value of a stock. At the end of a given day, the price is recorded. If the stock has gone up, the probability that it will go up tomorrow is 0.7. If the stock has gone down, the probability that it will go up tomorrow is only 0.5. Construct an appropriate transition matrix for the 2-state Markov process model for the fluctuating stock problem and using appropriate percentage variables and their definitions, formulate the model equations to determine the long-term percentage increase and decrease for the stock problem.
  (13 Marks)

## QUESTION 4 [40 MARKS]

- (a) A spring with a mass of 2 kg has natural length 0.5 m. A force of 25.6 N is required to maintain it stretched to a length of 0.7 m and then released with initial velocity 0, find the position of the mass at any time t, stating all physical laws to support the fundamental equations and associated concepts of your model and its solution before using the given data. (14 Marks)
- (b) Suppose a large lake that was formed by damming a river holds initially 100 million gallons of water. Because a nearby agricultural field was sprayed with a pesticide, the water has become contaminated. The concentration of the pesticide has been measured and is equal to 35ppm (parts per million), or  $35 \times 10^{-6}$ . The river continues to flow into the lake at a rate of 300 gal/min. The river is only slightly contaminated with a pesticide and has a concentration of 5 ppm. The flow of water over the dam can be controlled and is set at 400 gal/min. Assume that no additional spraying causes the lake to become even more contaminated. How long will it be before the water reaches an acceptable level of concentration equal to 15 ppm? (26 Marks)

**END OF QUESTION PAPER** 

**TOTAL MARKS = 150**