NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

Faculty of Health, Natural Resources and Applied Sciences

School of Natural and Applied Sciences

Department of Mathematics, Statistics and Actuarial Science

13 Jackson Kaujeua Street
Private Bag 13388
Windhoek
NAMIBIA

T: +264 61 207 2913
E: msas@nust.na
W: www.nust.na

QUALIFICATIONS : **BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS AND BACHELOR OF SCIENCE**	
QUALIFICATION CODES: **07BSAM, 07BSOC**	LEVEL: **6**
COURSE: **ORDINARY DIFFERENTIAL EQUATIONS**	COURSE CODE: **ODE602S**
DATE: **NOVEMBER 2024**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

FIRST OPPORTUNITY: QUESTION PAPER

EXAMINER: ***Prof Adetayo S. Eegunjobi***

MODERATOR: ***Prof Sunday A. Reju***

INSTRUCTIONS:

1. Answer **ANY FOUR (4)** questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS

1. Non-Programmable Calculator

ATTACHEMENTS

1. None

This paper consists of 3 pages including this front page

1. Discuss the existence and uniqueness of the following two IVPs and solve them.
   (a) $\frac{dy}{dx} = y^2(x) - 1, \quad y(0) = 0$ (7)
   (b) $4y'(x) = y^{\frac{1}{2}}, \quad y(0) = 0$ (7)
   (c) Solve
   $$y - xy'(x) = yy'(x) + x$$
   (11)

2. (a) The solutions of second order homogeneous differential equation of the form
   $$y''(x) + p(x)y'(x) + q(x)y(x) = f(x)$$
   are y_1 and y_2 where $p(x)$ and $q(x)$ are continuous on an open interval I, find the formula for $u(x)$ and $v(x)$ of the particular solution by using variation of parameters. (7)
   (b) Find $y_2(x)$ for all values of x if $y_1(x) = e^x \quad W(y_1, y_2) = e^x(x^2 - 2) \quad y_2(1) = 3$ (8)
   (c) Find the general solution of $x^2y''(x) - 2xy'(x) + 2y(x) = x^4e^x$ by using variation of parameter method (10)

3. (a) Solve $y'(x) - y(x)\tan x = -y(x)^2 \sec x$ (9)
   (b) Find the general solution of $y'(x) = 1 + (y - x)^2, \quad y_1(x) = x$ (8)
   (c) The quantity $N(t)$ of bacteria in a culture increased at a rate proportional to $N(t)$. The value of $N(t)$ was initially 100 and rose to 332 in one hour. What was the value of $N(t)$ after $\frac{3}{2}$ hours? (8)

4. (a) Using shifting with multiplication theorems, find the Laplace transform of $t^2e^{-8t}\sin t$. (8)
   (b) Evaluate
   $$\int_0^\infty e^{-2t}t^3 \sin t dt$$
   (7)
   (c) Find
   i.
   $$\mathcal{L}\left\{\frac{\cos 4t - \cos 5t}{t}\right\}$$
   (5)
   ii.
   $$\mathcal{L}\left\{\frac{\sin^2 t}{t}\right\}$$
   (5)

5. (a) Use Laplace transform to find $y''(t)+2y'(t)+5y(t) = e^{-t}\sin t, \quad y(0) = 0, \quad y'(0) = 1$ (8)

   (b) Find the general solution of $y''(x) + 6y'(x) + 9y(x) = 9x + 6$ (7)

   (c) Solve $xy'(x) + y(x) = x^4y^3(x)$ (10)

End of Exam!