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QUALIFICATION : **BACHELOR OF SCIENCE**	
QUALIFICATION CODE: **08BOSC**	LEVEL: **7**
COURSE: **QUANTUM PHYSICS**	COURSE CODE: **QPH702S**
DATE: **NOVEMBER 2024**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

SECOND OPPORTUNITY/SUPPLEMENTARY: QUESTION PAPER

EXAMINER:	Professor Dipti Ranjan Sahu
MODERATOR:	Professor Vijaya S. Vallabhapurapu

INSTRUCTIONS

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

This paper consists of 3 pages including this front page

QUESTION 1: **[20 MARKS]**

1.1 A particle has the wave function

$$\phi(r) = Ne^{-\alpha r}$$

where N is a normalization factor and α is a known real parameter.

1.1.1 Calculate the factor N. (3)

1.1.2 Calculate the expectation values <r> (2)

1.2 What is physical significance of wave function? (5)

1.3 Consider the Hamiltonian (10)

$$H = \frac{P_x{}^2 + P_y{}^2}{2m} + V(x, y) \rightarrow V(x, y) = \begin{cases} 0 & |x| < \frac{L}{2}, 0 < y < L \\ \infty & else \end{cases}$$

Determine the eigen value and normalized eigen function of the system

QUESTION 2: **[20 MARKS]**

A potential barrier is defined by:

$$V(x) = \begin{cases} 1.2 \text{ eV} & -\infty < x < -2 \\ 0 & -2 < x < 2 \\ 1.2 \text{ eV} & 2 < x < \infty \end{cases}$$

2.1 Sketch the graph of V(x) (2)

A particle of mass m and kinetic energy 1.0 eV is incident on this barrier from -∞.

2.1.1 Evaluate the acceptable wave function of the particle. (8)

2.1.2 Evaluate the conditions applicable at the boundaries (5)

2.2 Calculate the energy of the first excited quantum state of a particle in the two-dimensional potential $V(x, y) = \frac{1}{2} mw^2 (x^2 + 4y^2)$ (5)

QUESTION 3 **[20 MARKS]**

3.1 A particle is represented by the normalized wave function

$$\psi(x) = \begin{cases} \frac{\sqrt{15}(a^2 - x^2)}{4a^{\frac{5}{2}}} & \text{for } -a < x < a, \\ 0 & , otherwise \end{cases}$$

Determine, the uncertainty ΔP in its momentum (10)

3.2 The wave function for the ground state of a harmonic oscillator is

$$\psi_0(x) = C_0 e^{-\alpha^2 x^2/2}$$

find α and the energy corresponding to this state. (10)

QUESTION 4: **[20 MARKS]**

4.1 Evaluate the $[x, p_x]$ commutators and state the consequences of the results. (5)

4.2 What are Pauli spin matrices and what value of spin they correspond? (5)

4.3 Evaluate the matrix of L_x for $l = 1$ 10)

QUESTION 5: **[20 MARKS]**

5.1 Show that in the usual stationary state perturbation theory, if the Hamiltonian can be written $H = Ho + H'$ with $H_o\Phi_0 = E_o\Phi_0$, then the correction ΔE_0 is (10)

$$\Delta E_0 \approx \langle \Phi_0 | H' | \Phi_0 \rangle$$

5.2 Evaluate the eigenfunction and the energy of the state n = 1 for a quantum system with the

Potential energy V (x) =

$$\begin{cases} -0.5x & -\frac{L}{2} < x < 0 \\ 0.5x & 0 < x < \frac{L}{2} \end{cases}$$

using first order perturbation theory and the infinite potential well as the unperturbed system. Given $\Psi_1^o = \sqrt{\frac{2}{L}}\cos(\frac{\pi}{L}x)$

Useful Standard Integral

$$\int_{-\infty}^{\infty} e^{-y^2} dy = \sqrt{\pi}$$

$$\int_{-\infty}^{\infty} y^n e^{-y^2} dy = \frac{\sqrt{\pi}}{n}; \quad n \text{ even}; \qquad 0; \quad n \text{ odd}$$

$$\int_{-\infty}^{\infty} e^{-\alpha y^2} e^{-\beta y} dy = \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}} e^{\frac{\beta^2}{4\alpha}}$$

$$\int_0^{\infty} x^n e^{-x} dx = n!$$

Spherical harmonics $Y_l^m(\theta,\varphi) = \left[\frac{(2l+1)}{4\pi}\frac{(l-m)!}{(l+m)!}\right]^{\frac{1}{2}} e^{im\varphi} P_l^m(x)$

Associated Legendre polynomials: $P_l^m(x) = (-1)^l \frac{(1-x^2)^{m/2}}{2^l l!}\left(\frac{d}{dx}\right)^{l+m}(1-x^2)^l$, where $x = \cos\theta$

Radial eigenfunctions of hydrogen-like atoms:

$$R_{nl}(r) = \left(\frac{2Z}{a_0 n}\right)^{\frac{3}{2}}\left[\frac{(n-l-1)!}{2n[(n+l)!]^3}\right]^{\frac{1}{2}}\left(\frac{2Z}{a_0 n}r\right)^l e^{-\frac{Zr}{a_0 n}} L_{n-l-1}^{2l+1}(\rho)$$, where

$$L_{n-l-1}^{2l+1}(\rho) = \sum_{k=0}^{n-l-1}(-1)^k \cdot \frac{[(n+l)!]^2 \rho^k}{(n-l-1-k)!(2l+1+k)!k!},$$ and $\rho = \frac{2Z}{a_0 n}r$

END OF QUESTION PAPER