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| QUALIFICATION : BACHELOR OF SCIENCES IN APPLIED MATHEMATICS AND STATISTICS | |
|----------------------------------------------------------------------------|----------------------|
| QUALIFICATION CODE: 07BAMS | LEVEL: 5 |
| COURSE: STATISTICAL INFERENCE 1 | COURSE CODE: SIN502S |
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SUPPLEMENTARY/SECOND OPPORTUNITY: EXAMINATION QUESTION PAPER

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MODERATOR: DR D. NTIRAMPEBA

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

ATTACHEMENTS:

1. T- Table
2. Z-table
3. Chi-square table
4. F-table
5. U-table

This paper consists of 5 pages including this front page.

QUESTION 1**[20 MARKS]**

Write down the letter corresponding to the best answer for each question.

- 1.1 When the population is divided into mutually exclusive sets, and then a simple random sample is drawn from each set, this is called: [2]
A. Simple random sampling. B. Stratified random sampling.
C. Cluster random sampling. D. Systematic random sampling.
- 1.2 A marketing research firm divides the population of a state into geographic areas, and randomly selects some of the areas and takes a simple random sample of each selected area. This is an example of a [2]
A. Cluster random sample B. Systematic random sample
C. Simple random sample D. Stratified random sample.
- 1.3 The use of the laws of probability to make inferences and draw statistical conclusions about populations based on sample data is referred to as _____ [2]
A. Descriptive statistics B. Inferential statistics
C. Sample statistics D. Population statistics
- 1.4 A manufacturer of contact lenses is studying the curvature of the lenses it sells. In particular, the last 500 lenses sold had an average curvature of 0.5. The population is [2]
A. The 500 lenses.
B. 0.5.
C. The lenses sold today.
D. All the lenses sold by the manufacturer.
- 1.5 A political scientist is studying voters in California. It is appropriate for him to use a mean to describe: [2]
A. The age of a typical voter.
B. The party affiliation of a typical voter.
C. The sex of a typical voter.
D. The county of residence of a typical voter.
- 1.6 A researcher is studying students' behaviour in colleges in California. She takes a sample of 400 students from 10 colleges. The average age of all college students in California is? [2]
A. A statistic. B. A parameter.
C. The median. D. A population.

- 1.7 The standard deviation of a normal population is 10. You take a sample of 25 items from this population and compute a 95% confidence interval. To compute the confidence interval, you will use : [2]
- A. The t table because the degrees of freedom will be 24.
 - B. The t table because the sample standard deviation is known.
 - C. The z table because the population standard deviation is known.
 - D. The z table because the sample size is small.
- 1.8 If in a random sample of 400 items, 88 are found to be defective. If the null hypothesis is that 20% of the items in the population are defective, what is the value of the test statistic? [2]
- A. 0.02
 - B. 1
 - C. 0.9656
 - D. 1.03
- 1.9 A 92% confidence interval for population proportion is 32.4% to 47.6%, the value of sample proportion is: [2]
- A. 40%
 - B. 32.4%
 - C. 47.6%
 - D. 80%
- 1.10 In a simple random survey of 89 teachers of high school AP Statistics, 73 said that it was the most satisfying, most enjoyable course they had ever taught. Establish a 98% confidence interval estimate of the proportion of all high school AP Statistics teachers who feel this way. [2]
- A. 0.820 ± 0.004
 - B. 0.820 ± 0.041
 - C. 0.820 ± 0.084
 - D. 0.820 ± 0.095

QUESTION 2

[44 MARKS]

- 2.1 The lifetime of a light bulb is normally distributed with the mean 3000 hours and a standard deviation of 696 hours. A simple random sample of 36 bulbs is taken.
- (a) What is the expected value, standard deviation, and shape of the sampling distribution of \bar{x} ? [3]
 - (b) What is the probability that the average lifetime in the sample will be between 2670.56 and 2809.76 hours? [5]
 - (c) How large of a sample needs to be taken to provide a 0.01 probability that the average lifetime in the sample will be equal to or greater than 3219.24 hours? [5]

- 2.2 The personnel department of a large corporation wants to estimate the family dental expenses of its employees to determine the feasibility of providing dental insurance plan. A random sample of 10 employees reveals the following family dental expenses (in N\$) for the past year.

110 362 246 85 510 208 173 425 316 179

- (a) Find the mean point estimate for the employees' dental expenditure in the past year. [3]
 - (b) Compute a 95% confidence interval for the true population mean in the employees' dental expenditure. [5]
- 2.3 A study was conducted to investigate the effectiveness of hypnotism in reducing pain. Results for randomly selected subjects are shown in the table.

| | | | | | | | | |
|--------|-----|-----|-----|------|------|-----|-----|------|
| Before | 6.6 | 6.5 | 9.0 | 10.3 | 11.3 | 8.1 | 6.3 | 11.6 |
| After | 6.8 | 2.4 | 7.4 | 8.5 | 8.1 | 6.1 | 3.4 | 2.0 |

At a 5% level of significance, from the sample data, is there sufficient evidence to conclude that the sensory measurements, on average, are lower after hypnotism? Conduct an appropriate hypothesis test. [10]

- 2.4 With individual lines at its various windows, a post-office is interested in the standard deviation for normally distributed waiting times for customers on Friday. The post-office experiments with a single main waiting line and find that for a random sample of 25 customers, the waiting times for customers have a variance of 12.25 minutes.
- (a) With a significance level of 5%, construct a confidence interval estimate for the variance waiting times of all customers at this post-office on a Friday. [6]
 - (b) Assuming that the estimated population variance at this post-office is 51.84 minutes, is there evidence at 1% level of significance to conclude that a single main waiting line causes lower variation among waiting times? [7]

QUESTION 3**[36 MARKS]**

- 3.1 The contingency table below shows a random sample of 500 U. S adults who were questioned regarding their political affiliation and opinion on a tax reform bill.

| Political Affiliation | Opinion on Tax Reform | | |
|-----------------------|-----------------------|-------------|--------|
| | Favor | Indifferent | Oppose |
| Democrat | 138 | 83 | 64 |
| Republican | 64 | 67 | 84 |

Test if the political affiliation and their opinion on a tax reform bill are dependent at a 5% level of significance. [13]

- 3.2 Someone has told you that men are better in abstract reasoning than women. You are sceptical, so you decide to test this idea. You randomly select eight adult men and eight adult women living in your hometown and administer an abstract reasoning test. A higher score reflects better abstract reasoning abilities. You obtain the following scores:

| | | | | | | | | |
|-------|----|----|----|----|----|----|----|----|
| Men | 70 | 86 | 60 | 92 | 82 | 50 | 74 | 94 |
| Women | 81 | 80 | 50 | 95 | 93 | 65 | 90 | 75 |

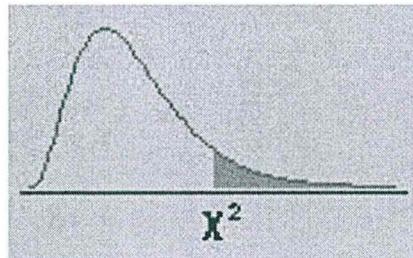
At the 5% level of significance, use the Mann-Whitney U test to test if there is a difference in scores between the two groups. [8]

- 3.3 Students were given different drug treatments before revising for their exams. Some were given a memory drug, some a placebo drug and some no treatment. The exam scores (%) are shown below for the three different groups:

| Memory drug | Placebo | No treatment |
|-------------|---------|--------------|
| 70 | 37 | 30 |
| 77 | 43 | 10 |
| 83 | 50 | 17 |

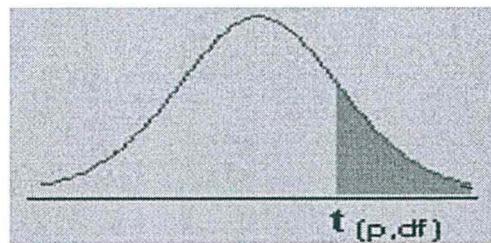
Test the hypothesis that the treatments had different effects. Use alpha = 0.05. [15]

APPENDIX E: The Chi-Square Distribution



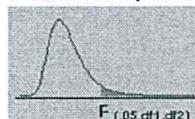
| df\p | .995 | .990 | .975 | .950 | .900 | .750 | .500 | .250 | .100 | .050 | .025 | .010 | .005 |
|------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1 | 0.00004 | 0.00016 | 0.00098 | 0.00393 | 0.01579 | 0.10153 | 0.45494 | 1.32330 | 2.70554 | 3.84146 | 5.02389 | 6.63490 | 7.87944 |
| 2 | 0.01003 | 0.02010 | 0.05064 | 0.10259 | 0.21072 | 0.57536 | 1.38629 | 2.77259 | 4.60517 | 5.99146 | 7.37776 | 9.21034 | 10.59663 |
| 3 | 0.07172 | 0.11483 | 0.21580 | 0.35185 | 0.58437 | 1.21253 | 2.36597 | 4.10834 | 6.25139 | 7.81473 | 9.34840 | 11.34487 | 12.83816 |
| 4 | 0.20699 | 0.29711 | 0.48442 | 0.71072 | 1.06362 | 1.92256 | 3.35669 | 5.38527 | 7.77944 | 9.48773 | 11.14329 | 13.27670 | 14.86026 |
| 5 | 0.41174 | 0.55430 | 0.83121 | 1.14548 | 1.61031 | 2.67460 | 4.35146 | 6.62568 | 9.23636 | 11.07050 | 12.83250 | 15.08627 | 16.74960 |
| 6 | 0.67573 | 0.87209 | 1.23734 | 1.63538 | 2.20413 | 3.45460 | 5.34812 | 7.84080 | 10.64464 | 12.59159 | 14.44938 | 16.81189 | 18.54758 |
| 7 | 0.98926 | 1.23904 | 1.68987 | 2.16735 | 2.83311 | 4.25485 | 6.34581 | 9.03715 | 12.01704 | 14.06714 | 16.01276 | 18.47531 | 20.27774 |
| 8 | 1.34441 | 1.64650 | 2.17973 | 2.73264 | 3.48954 | 5.07064 | 7.34412 | 10.21885 | 13.36157 | 15.50731 | 17.53455 | 20.09024 | 21.95495 |
| 9 | 1.73493 | 2.08790 | 2.70039 | 3.32511 | 4.16816 | 5.89883 | 8.34283 | 11.38875 | 14.68366 | 16.91898 | 19.02277 | 21.66599 | 23.58935 |
| 10 | 2.15586 | 2.55821 | 3.24697 | 3.94030 | 4.86518 | 6.73720 | 9.34182 | 12.54886 | 15.98718 | 18.30704 | 20.48318 | 23.20925 | 25.18818 |
| 11 | 2.60322 | 3.05348 | 3.81575 | 4.57481 | 5.57778 | 7.58414 | 10.34100 | 13.70069 | 17.27501 | 19.67514 | 21.92005 | 24.72497 | 26.75685 |
| 12 | 3.07382 | 3.57057 | 4.40379 | 5.22603 | 6.30380 | 8.43842 | 11.34032 | 14.84540 | 18.54935 | 21.02607 | 23.33666 | 26.21697 | 28.29952 |
| 13 | 3.56503 | 4.10692 | 5.00875 | 5.89186 | 7.04150 | 9.29907 | 12.33976 | 15.98391 | 19.81193 | 22.36203 | 24.73560 | 27.68825 | 29.81947 |
| 14 | 4.07467 | 4.66043 | 5.62873 | 6.57063 | 7.78953 | 10.16531 | 13.33927 | 17.11693 | 21.06414 | 23.68479 | 26.11895 | 29.14124 | 31.31935 |
| 15 | 4.60092 | 5.22935 | 6.26214 | 7.26094 | 8.54676 | 11.03654 | 14.33886 | 18.24509 | 22.30713 | 24.99579 | 27.48839 | 30.57791 | 32.80132 |
| 16 | 5.14221 | 5.81221 | 6.90766 | 7.96165 | 9.31224 | 11.91222 | 15.33850 | 19.36886 | 23.54183 | 26.29623 | 28.84535 | 31.99993 | 34.26719 |
| 17 | 5.69722 | 6.40776 | 7.56419 | 8.67176 | 10.08519 | 12.79193 | 16.33818 | 20.48868 | 24.76904 | 27.58711 | 30.19101 | 33.40866 | 35.71847 |
| 18 | 6.26480 | 7.01491 | 8.23075 | 9.39046 | 10.86494 | 13.67529 | 17.33790 | 21.60489 | 25.98942 | 28.86930 | 31.52638 | 34.80531 | 37.15645 |
| 19 | 6.84397 | 7.63273 | 8.90652 | 10.11701 | 11.65091 | 14.56200 | 18.33765 | 22.71781 | 27.20357 | 30.14353 | 32.85233 | 36.19087 | 38.58226 |
| 20 | 7.43384 | 8.26040 | 9.59078 | 10.85081 | 12.44261 | 15.45177 | 19.33743 | 23.82769 | 28.41198 | 31.41043 | 34.16961 | 37.56623 | 39.99685 |
| 21 | 8.03365 | 8.89720 | 10.28290 | 11.59131 | 13.23960 | 16.34438 | 20.33723 | 24.93478 | 29.61509 | 32.67057 | 35.47888 | 38.93217 | 41.40106 |
| 22 | 8.64272 | 9.54249 | 10.98232 | 12.33801 | 14.04149 | 17.23962 | 21.33704 | 26.03927 | 30.81328 | 33.92444 | 36.78071 | 40.28936 | 42.79565 |
| 23 | 9.26042 | 10.19572 | 11.68855 | 13.09051 | 14.84796 | 18.13730 | 22.33688 | 27.14134 | 32.00690 | 35.17246 | 38.07563 | 41.63840 | 44.18128 |
| 24 | 9.88623 | 10.85636 | 12.40115 | 13.84843 | 15.65868 | 19.03725 | 23.33673 | 28.24115 | 33.19624 | 36.41503 | 39.36408 | 42.97982 | 45.55851 |
| 25 | 10.51965 | 11.52398 | 13.11972 | 14.61141 | 16.47341 | 19.93934 | 24.33659 | 29.33885 | 34.38159 | 37.65248 | 40.64647 | 44.31410 | 46.92789 |
| 26 | 11.16024 | 12.19815 | 13.84390 | 15.37916 | 17.29188 | 20.84343 | 25.33646 | 30.43457 | 35.56317 | 38.88514 | 41.92317 | 45.64168 | 48.28988 |
| 27 | 11.80759 | 12.87850 | 14.57338 | 16.15140 | 18.11390 | 21.74940 | 26.33634 | 31.52841 | 36.74122 | 40.11327 | 43.19451 | 46.96294 | 49.64492 |
| 28 | 12.46134 | 13.56471 | 15.30786 | 16.92788 | 18.93924 | 22.65716 | 27.33623 | 32.62049 | 37.91592 | 41.33714 | 44.46079 | 48.27824 | 50.99338 |
| 29 | 13.12115 | 14.25645 | 16.04707 | 17.70837 | 19.76774 | 23.56659 | 28.33613 | 33.71091 | 39.08747 | 42.55697 | 45.72229 | 49.58788 | 52.33562 |
| 30 | 13.78672 | 14.95346 | 16.79077 | 18.49266 | 20.59923 | 24.47761 | 29.33603 | 34.79974 | 40.25602 | 43.77297 | 46.97924 | 50.89218 | 53.67196 |

APPENDIX D: The t-distribution



| df\p | 0.40 | 0.25 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.0005 |
|-------------|-------------|-------------|-------------|-------------|--------------|-------------|--------------|---------------|
| 1 | 0.324920 | 1.000000 | 3.077684 | 6.313752 | 12.70620 | 31.82052 | 63.65674 | 636.6192 |
| 2 | 0.288675 | 0.816497 | 1.885618 | 2.919986 | 4.30265 | 6.96456 | 9.92484 | 31.5991 |
| 3 | 0.276671 | 0.764892 | 1.637744 | 2.353363 | 3.18245 | 4.54070 | 5.84091 | 12.9240 |
| 4 | 0.270722 | 0.740697 | 1.533206 | 2.131847 | 2.77645 | 3.74695 | 4.60409 | 8.6103 |
| 5 | 0.267181 | 0.726687 | 1.475884 | 2.015048 | 2.57058 | 3.36493 | 4.03214 | 6.8688 |
| 6 | 0.264835 | 0.717558 | 1.439756 | 1.943180 | 2.44691 | 3.14267 | 3.70743 | 5.9588 |
| 7 | 0.263167 | 0.711142 | 1.414924 | 1.894579 | 2.36462 | 2.99795 | 3.49948 | 5.4079 |
| 8 | 0.261921 | 0.706387 | 1.396815 | 1.859548 | 2.30600 | 2.89646 | 3.35539 | 5.0413 |
| 9 | 0.260955 | 0.702722 | 1.383029 | 1.833113 | 2.26216 | 2.82144 | 3.24984 | 4.7809 |
| 10 | 0.260185 | 0.699812 | 1.372184 | 1.812461 | 2.22814 | 2.76377 | 3.16927 | 4.5869 |
| 11 | 0.259556 | 0.697445 | 1.363430 | 1.795885 | 2.20099 | 2.71808 | 3.10581 | 4.4370 |
| 12 | 0.259033 | 0.695483 | 1.356217 | 1.782288 | 2.17881 | 2.68100 | 3.05454 | 4.3178 |
| 13 | 0.258591 | 0.693829 | 1.350171 | 1.770933 | 2.16037 | 2.65031 | 3.01228 | 4.2208 |
| 14 | 0.258213 | 0.692417 | 1.345030 | 1.761310 | 2.14479 | 2.62449 | 2.97684 | 4.1405 |
| 15 | 0.257885 | 0.691197 | 1.340606 | 1.753050 | 2.13145 | 2.60248 | 2.94671 | 4.0728 |
| 16 | 0.257599 | 0.690132 | 1.336757 | 1.745884 | 2.11991 | 2.58349 | 2.92078 | 4.0150 |
| 17 | 0.257347 | 0.689195 | 1.333379 | 1.739607 | 2.10982 | 2.56693 | 2.89823 | 3.9651 |
| 18 | 0.257123 | 0.688364 | 1.330391 | 1.734064 | 2.10092 | 2.55238 | 2.87844 | 3.9216 |
| 19 | 0.256923 | 0.687621 | 1.327728 | 1.729133 | 2.09302 | 2.53948 | 2.86093 | 3.8834 |
| 20 | 0.256743 | 0.686954 | 1.325341 | 1.724718 | 2.08596 | 2.52798 | 2.84534 | 3.8495 |
| 21 | 0.256580 | 0.686352 | 1.323188 | 1.720743 | 2.07961 | 2.51765 | 2.83136 | 3.8193 |
| 22 | 0.256432 | 0.685805 | 1.321237 | 1.717144 | 2.07387 | 2.50832 | 2.81876 | 3.7921 |
| 23 | 0.256297 | 0.685306 | 1.319460 | 1.713872 | 2.06866 | 2.49987 | 2.80734 | 3.7676 |
| 24 | 0.256173 | 0.684850 | 1.317836 | 1.710882 | 2.06390 | 2.49216 | 2.79694 | 3.7454 |
| 25 | 0.256060 | 0.684430 | 1.316345 | 1.708141 | 2.05954 | 2.48511 | 2.78744 | 3.7251 |
| 26 | 0.255955 | 0.684043 | 1.314972 | 1.705618 | 2.05553 | 2.47863 | 2.77871 | 3.7066 |
| 27 | 0.255858 | 0.683685 | 1.313703 | 1.703288 | 2.05183 | 2.47266 | 2.77068 | 3.6896 |
| 28 | 0.255768 | 0.683353 | 1.312527 | 1.701131 | 2.04841 | 2.46714 | 2.76326 | 3.6739 |
| 29 | 0.255684 | 0.683044 | 1.311434 | 1.699127 | 2.04523 | 2.46202 | 2.75639 | 3.6594 |
| 30 | 0.255605 | 0.682756 | 1.310415 | 1.697261 | 2.04227 | 2.45726 | 2.75000 | 3.6460 |
| inf | 0.253347 | 0.674490 | 1.281552 | 1.644854 | 1.95996 | 2.32635 | 2.57583 | 3.2905 |

F Table for alpha=0.05

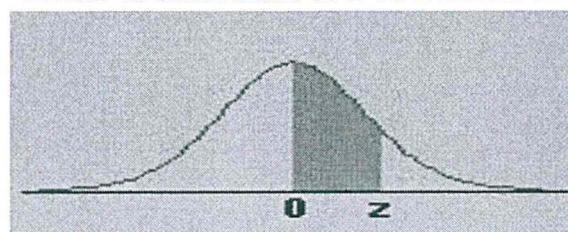


| df2/df1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 | 20 | 24 | 30 | 40 | 60 | 120 | INF |
|---------|----------|--------|----------|----------|----------|---------|----------|----------|----------|----------|---------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1 | 161.4476 | 199.5 | 215.7073 | 224.5832 | 230.1619 | 233.986 | 236.7684 | 238.8827 | 240.5433 | 241.8817 | 243.906 | 245.9499 | 248.0131 | 249.0518 | 250.0951 | 251.1432 | 252.1957 | 253.2529 | 254.3144 |
| 2 | 18.5128 | 19 | 19.1643 | 19.2468 | 19.2964 | 19.3295 | 19.3532 | 19.371 | 19.3848 | 19.3959 | 19.4125 | 19.4291 | 19.4458 | 19.4541 | 19.4624 | 19.4707 | 19.4791 | 19.4874 | 19.4957 |
| 3 | 10.128 | 9.5521 | 9.2766 | 9.1172 | 9.0135 | 8.9406 | 8.8867 | 8.8452 | 8.8123 | 8.7855 | 8.7446 | 8.7029 | 8.6602 | 8.6385 | 8.6166 | 8.5944 | 8.572 | 8.5494 | 8.5264 |
| 4 | 7.7086 | 6.9443 | 6.5914 | 6.3882 | 6.2561 | 6.1631 | 6.0942 | 6.041 | 5.9988 | 5.9644 | 5.9117 | 5.8578 | 5.8025 | 5.7744 | 5.7459 | 5.717 | 5.6877 | 5.6581 | 5.6281 |
| 5 | 6.6079 | 5.7861 | 5.4095 | 5.1922 | 5.0503 | 4.9503 | 4.8759 | 4.8183 | 4.7725 | 4.7351 | 4.6777 | 4.6188 | 4.5581 | 4.5272 | 4.4957 | 4.4638 | 4.4314 | 4.3985 | 4.365 |
| 6 | 5.9874 | 5.1433 | 4.7571 | 4.5337 | 4.3874 | 4.2839 | 4.2067 | 4.1468 | 4.099 | 4.06 | 3.9999 | 3.9381 | 3.8742 | 3.8415 | 3.8082 | 3.7743 | 3.7398 | 3.7047 | 3.6689 |
| 7 | 5.5914 | 4.7374 | 4.3468 | 4.1203 | 3.9715 | 3.866 | 3.787 | 3.7257 | 3.6767 | 3.6365 | 3.5747 | 3.5107 | 3.4445 | 3.4105 | 3.3758 | 3.3404 | 3.3043 | 3.2674 | 3.2298 |
| 8 | 5.3177 | 4.459 | 4.0662 | 3.8379 | 3.6875 | 3.5806 | 3.5005 | 3.4381 | 3.3881 | 3.3472 | 3.2839 | 3.2184 | 3.1503 | 3.1152 | 3.0794 | 3.0428 | 3.0053 | 2.9669 | 2.9276 |
| 9 | 5.1174 | 4.2565 | 3.8625 | 3.6331 | 3.4817 | 3.3738 | 3.2927 | 3.2296 | 3.1789 | 3.1373 | 3.0729 | 3.0061 | 2.9365 | 2.9005 | 2.8637 | 2.8259 | 2.7872 | 2.7475 | 2.7067 |
| 10 | 4.9646 | 4.1028 | 3.7083 | 3.478 | 3.3258 | 3.2172 | 3.1355 | 3.0717 | 3.0204 | 2.9782 | 2.913 | 2.845 | 2.774 | 2.7372 | 2.6996 | 2.6609 | 2.6211 | 2.5801 | 2.5379 |
| 11 | 4.8443 | 3.9823 | 3.5874 | 3.3567 | 3.2039 | 3.0946 | 3.0123 | 2.948 | 2.8962 | 2.8536 | 2.7876 | 2.7186 | 2.6464 | 2.609 | 2.5705 | 2.5309 | 2.4901 | 2.448 | 2.4045 |
| 12 | 4.7472 | 3.8853 | 3.4903 | 3.2592 | 3.1059 | 2.9961 | 2.9134 | 2.8486 | 2.7964 | 2.7534 | 2.6866 | 2.6169 | 2.5436 | 2.5055 | 2.4663 | 2.4259 | 2.3842 | 2.341 | 2.2962 |
| 13 | 4.6672 | 3.8056 | 3.4105 | 3.1791 | 3.0254 | 2.9153 | 2.8321 | 2.7669 | 2.7144 | 2.671 | 2.6037 | 2.5331 | 2.4589 | 2.4202 | 2.3803 | 2.3392 | 2.2966 | 2.2524 | 2.2064 |
| 14 | 4.6001 | 3.7389 | 3.3439 | 3.1122 | 2.9582 | 2.8477 | 2.7642 | 2.6987 | 2.6458 | 2.6022 | 2.5342 | 2.463 | 2.3879 | 2.3487 | 2.3082 | 2.2664 | 2.2229 | 2.1778 | 2.1307 |
| 15 | 4.5431 | 3.6823 | 3.2874 | 3.0556 | 2.9013 | 2.7905 | 2.7066 | 2.6408 | 2.5876 | 2.5437 | 2.4753 | 2.4034 | 2.3275 | 2.2878 | 2.2468 | 2.2043 | 2.1601 | 2.1141 | 2.0658 |
| 16 | 4.494 | 3.6337 | 3.2389 | 3.0069 | 2.8524 | 2.7413 | 2.6572 | 2.5911 | 2.5377 | 2.4935 | 2.4247 | 2.3522 | 2.2756 | 2.2354 | 2.1938 | 2.1507 | 2.1058 | 2.0589 | 2.0096 |
| 17 | 4.4513 | 3.5915 | 3.1968 | 2.9647 | 2.81 | 2.6987 | 2.6143 | 2.548 | 2.4943 | 2.4499 | 2.3807 | 2.3077 | 2.2304 | 2.1898 | 2.1477 | 2.104 | 2.0584 | 2.0107 | 1.9604 |
| 18 | 4.4139 | 3.5546 | 3.1599 | 2.9277 | 2.7729 | 2.6613 | 2.5767 | 2.5102 | 2.4563 | 2.4117 | 2.3421 | 2.2686 | 2.1906 | 2.1497 | 2.1071 | 2.0629 | 2.0166 | 1.9681 | 1.9168 |
| 19 | 4.3807 | 3.5219 | 3.1274 | 2.8951 | 2.7401 | 2.6283 | 2.5435 | 2.4768 | 2.4227 | 2.3779 | 2.308 | 2.2341 | 2.1555 | 2.1141 | 2.0712 | 2.0264 | 1.9795 | 1.9302 | 1.878 |
| 20 | 4.3512 | 3.4928 | 3.0984 | 2.8661 | 2.7109 | 2.599 | 2.514 | 2.4471 | 2.3928 | 2.3479 | 2.2776 | 2.2033 | 2.1242 | 2.0825 | 2.0391 | 1.9938 | 1.9464 | 1.8963 | 1.8432 |
| 21 | 4.3248 | 3.4668 | 3.0725 | 2.8401 | 2.6848 | 2.5727 | 2.4876 | 2.4205 | 2.366 | 2.321 | 2.2504 | 2.1757 | 2.096 | 2.054 | 2.0102 | 1.9645 | 1.9165 | 1.8657 | 1.8117 |
| 22 | 4.3009 | 3.4434 | 3.0491 | 2.8167 | 2.6613 | 2.5491 | 2.4638 | 2.3965 | 2.3419 | 2.2967 | 2.2258 | 2.1508 | 2.0707 | 2.0283 | 1.9842 | 1.938 | 1.8894 | 1.838 | 1.7831 |
| 23 | 4.2793 | 3.4221 | 3.028 | 2.7955 | 2.64 | 2.5277 | 2.4422 | 2.3748 | 2.3201 | 2.2747 | 2.2036 | 2.1282 | 2.0476 | 2.005 | 1.9605 | 1.9139 | 1.8648 | 1.8128 | 1.757 |
| 24 | 4.2597 | 3.4028 | 3.0088 | 2.7763 | 2.6207 | 2.5082 | 2.4226 | 2.3551 | 2.3002 | 2.2547 | 2.1834 | 2.1077 | 2.0267 | 1.9838 | 1.939 | 1.892 | 1.8424 | 1.7896 | 1.733 |
| 25 | 4.2417 | 3.3852 | 2.9912 | 2.7587 | 2.603 | 2.4904 | 2.4047 | 2.3371 | 2.2821 | 2.2365 | 2.1649 | 2.0889 | 2.0075 | 1.9643 | 1.9192 | 1.8718 | 1.8217 | 1.7684 | 1.711 |
| 26 | 4.2252 | 3.369 | 2.9752 | 2.7426 | 2.5868 | 2.4741 | 2.3883 | 2.3205 | 2.2655 | 2.2197 | 2.1479 | 2.0716 | 1.9898 | 1.9464 | 1.901 | 1.8533 | 1.8027 | 1.7488 | 1.6906 |
| 27 | 4.21 | 3.3541 | 2.9604 | 2.7278 | 2.5719 | 2.4591 | 2.3732 | 2.3053 | 2.2501 | 2.2043 | 2.1323 | 2.0558 | 1.9736 | 1.9299 | 1.8842 | 1.8361 | 1.7851 | 1.7306 | 1.6717 |
| 28 | 4.196 | 3.3404 | 2.9467 | 2.7141 | 2.5581 | 2.4453 | 2.3593 | 2.2913 | 2.236 | 2.19 | 2.1179 | 2.0411 | 1.9586 | 1.9147 | 1.8687 | 1.8203 | 1.7689 | 1.7138 | 1.6541 |
| 29 | 4.183 | 3.3277 | 2.934 | 2.7014 | 2.5454 | 2.4324 | 2.3463 | 2.2783 | 2.2229 | 2.1768 | 2.1045 | 2.0275 | 1.9446 | 1.9005 | 1.8543 | 1.8055 | 1.7537 | 1.6981 | 1.6376 |
| 30 | 4.1709 | 3.3158 | 2.9223 | 2.6896 | 2.5336 | 2.4205 | 2.3343 | 2.2662 | 2.2107 | 2.1646 | 2.0921 | 2.0148 | 1.9317 | 1.8874 | 1.8409 | 1.7918 | 1.7396 | 1.6835 | 1.6223 |
| 40 | 4.0847 | 3.2317 | 2.8387 | 2.606 | 2.4495 | 2.3359 | 2.249 | 2.1802 | 2.124 | 2.0772 | 2.0035 | 1.9245 | 1.8389 | 1.7929 | 1.7444 | 1.6928 | 1.6373 | 1.5766 | 1.5089 |
| 60 | 4.0012 | 3.1504 | 2.7581 | 2.5252 | 2.3683 | 2.2541 | 2.1665 | 2.097 | 2.0401 | 1.9926 | 1.9174 | 1.8364 | 1.748 | 1.7001 | 1.6491 | 1.5943 | 1.5343 | 1.4673 | 1.3893 |
| 120 | 3.9201 | 3.0718 | 2.6802 | 2.4472 | 2.2899 | 2.175 | 2.0868 | 2.0164 | 1.9588 | 1.9105 | 1.8337 | 1.7505 | 1.6587 | 1.6084 | 1.5543 | 1.4952 | 1.429 | 1.3519 | 1.2539 |
| inf | 3.8415 | 2.9957 | 2.6049 | 2.3719 | 2.2141 | 2.0986 | 2.0096 | 1.9384 | 1.8799 | 1.8307 | 1.7522 | 1.6664 | 1.5705 | 1.5173 | 1.4591 | 1.394 | 1.318 | 1.2214 | 1 |

Critical Values of the Mann-Whitney U
 (Two-Tailed Testing)

| n ₂ | α | n ₁ | | | | | | | | | | | | | | | | | | |
|----------------|----------|----------------|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|--|
| | | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | |
| 3 | .05 | -- | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 7 | 8 | |
| | .01 | -- | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | |
| 4 | .05 | -- | 0 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 13 | 14 | |
| | .01 | -- | -- | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 | 6 | 6 | 7 | 8 | |
| 5 | .05 | 0 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 11 | 12 | 13 | 14 | 15 | 17 | 18 | 19 | 20 | |
| | .01 | -- | -- | 0 | 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | |
| 6 | .05 | 1 | 2 | 3 | 5 | 6 | 8 | 10 | 11 | 13 | 14 | 16 | 17 | 19 | 21 | 22 | 24 | 25 | 27 | |
| | .01 | -- | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 10 | 11 | 12 | 13 | 15 | 16 | 17 | 18 | |
| 7 | .05 | 1 | 3 | 5 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | |
| | .01 | -- | 0 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 12 | 13 | 15 | 16 | 18 | 19 | 21 | 22 | 24 | |
| 8 | .05 | 2 | 4 | 6 | 8 | 10 | 13 | 15 | 17 | 19 | 22 | 24 | 26 | 29 | 31 | 34 | 36 | 38 | 41 | |
| | .01 | -- | 1 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | |
| 9 | .05 | 2 | 4 | 7 | 10 | 12 | 15 | 17 | 20 | 23 | 26 | 28 | 31 | 34 | 37 | 39 | 42 | 45 | 48 | |
| | .01 | 0 | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 16 | 18 | 20 | 22 | 24 | 27 | 29 | 31 | 33 | 36 | |
| 10 | .05 | 3 | 5 | 8 | 11 | 14 | 17 | 20 | 23 | 26 | 29 | 33 | 36 | 39 | 42 | 45 | 48 | 52 | 55 | |
| | .01 | 0 | 2 | 4 | 6 | 9 | 11 | 13 | 16 | 18 | 21 | 24 | 26 | 29 | 31 | 34 | 37 | 39 | 42 | |
| 11 | .05 | 3 | 6 | 9 | 13 | 16 | 19 | 23 | 26 | 30 | 33 | 37 | 40 | 44 | 47 | 51 | 55 | 58 | 62 | |
| | .01 | 0 | 2 | 5 | 7 | 10 | 13 | 16 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 | 48 | |
| 12 | .05 | 4 | 7 | 11 | 14 | 18 | 22 | 26 | 29 | 33 | 37 | 41 | 45 | 49 | 53 | 57 | 61 | 65 | 69 | |
| | .01 | 1 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 31 | 34 | 37 | 41 | 44 | 47 | 51 | 54 | |
| 13 | .05 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 33 | 37 | 41 | 45 | 50 | 54 | 59 | 63 | 67 | 72 | 76 | |
| | .01 | 1 | 3 | 7 | 10 | 13 | 17 | 20 | 24 | 27 | 31 | 34 | 38 | 42 | 45 | 49 | 53 | 56 | 60 | |
| 14 | .05 | 5 | 9 | 13 | 17 | 22 | 26 | 31 | 36 | 40 | 45 | 50 | 55 | 59 | 64 | 67 | 74 | 78 | 83 | |
| | .01 | 1 | 4 | 7 | 11 | 15 | 18 | 22 | 26 | 30 | 34 | 38 | 42 | 46 | 50 | 54 | 58 | 63 | 67 | |
| 15 | .05 | 5 | 10 | 14 | 19 | 24 | 29 | 34 | 39 | 44 | 49 | 54 | 59 | 64 | 70 | 75 | 80 | 85 | 90 | |
| | .01 | 2 | 5 | 8 | 12 | 16 | 20 | 24 | 29 | 33 | 37 | 42 | 46 | 51 | 55 | 60 | 64 | 69 | 73 | |
| 16 | .05 | 6 | 11 | 15 | 21 | 26 | 31 | 37 | 42 | 47 | 53 | 59 | 64 | 70 | 75 | 81 | 86 | 92 | 98 | |
| | .01 | 2 | 5 | 9 | 13 | 18 | 22 | 27 | 31 | 36 | 41 | 45 | 50 | 55 | 60 | 65 | 70 | 74 | 79 | |
| 17 | .05 | 6 | 11 | 17 | 22 | 28 | 34 | 39 | 45 | 51 | 57 | 63 | 67 | 75 | 81 | 87 | 93 | 99 | 105 | |
| | .01 | 2 | 6 | 10 | 15 | 19 | 24 | 29 | 34 | 39 | 44 | 49 | 54 | 60 | 65 | 70 | 75 | 81 | 86 | |
| 18 | .05 | 7 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 55 | 61 | 67 | 74 | 80 | 86 | 93 | 99 | 106 | 112 | |
| | .01 | 2 | 6 | 11 | 16 | 21 | 26 | 31 | 37 | 42 | 47 | 53 | 58 | 64 | 70 | 75 | 81 | 87 | 92 | |
| 19 | .05 | 7 | 13 | 19 | 25 | 32 | 38 | 45 | 52 | 58 | 65 | 72 | 78 | 85 | 92 | 99 | 106 | 113 | 119 | |
| | .01 | 3 | 7 | 12 | 17 | 22 | 28 | 33 | 39 | 45 | 51 | 56 | 63 | 69 | 74 | 81 | 87 | 93 | 99 | |
| 20 | .05 | 8 | 14 | 20 | 27 | 34 | 41 | 48 | 55 | 62 | 69 | 76 | 83 | 90 | 98 | 105 | 112 | 119 | 127 | |
| | .01 | 3 | 8 | 13 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 67 | 73 | 79 | 86 | 92 | 99 | 105 | |

APPENDIX C: The Standard Normal Distribution



| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |