NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 35BAMS	**LEVEL:** 6
COURSE CODE: NUM701S	**COURSE NAME:** NUMERICAL METHODS 1
SESSION: JULY 2022	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr S.N. NEOSSI NGUETCHUE
MODERATOR:	Prof S.S. MOTSA

INSTRUCTIONS
1. Answer ALL the questions in the booklet provided. 2. Show clearly all the steps used in the calculations. All numerical results must be given using 4 decimals where necessary unless mentioned otherwise. 3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Attachments

None

Problem 1. [21 marks]

1-1-1. Why is the nested form of a polynomial important compared to its canonical (original) form? Give an example illustrating your statement with the number of operations involved (you can use a third degree polynomial of your choice). [2+2=4]

1-1-2. Write down a pseudo-code that uses the nested form of a polynomial of degree n and evaluates it at $x = x_0$. [3]

1-2. Write down the general formula of the Taylor's expansion (with integral remainder) of a function $f(x)$ about $x = x_0$. [5]

1-3 The nth root of the number N can be found by solving the equation $x^n - N = 0$.

1-3-1 For the above equation, show that Newton's method gives: [5]

$$x_{i+1} = \frac{1}{n}\left[(n-1)x_i + \frac{N}{x_i^{n-1}}\right]$$

1-3-2 Use the above result to find $(161)^{1/3}$ after three iterations with $x_0 = 6.0$ as the starting point. [4]

Problem 2 [30 marks]

2-1. Write down in details the formulae of the Lagrange and Newton's form of the polynomial that interpolates the set of data points $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$. [7]

2-2. Use the results in **2-1.** to determine the Lagrange and Newton's form of the polynomial that interpolates the data set $(0, 2), (1, 5)$ and $(2, 12)$. [18]

2-3. If an extra point say $(4, 9)$ is to be added to the above data set, which of the two forms in **2-1.** would be more efficient and why? [Don't compute the corresponding polynomials.] [5]

Problem 3. [30 marks]

3-1. Determine the error term for the formula [5]

$$f'(x) \approx \frac{1}{2h}[4f(x+h) - 3f(x) - f(x+2h)]$$

3-2. Use the above formula to approximate $f'(1.8)$ with $f(x) = \ln x$ using $h = 0.1,\ 0.01$ and 0.001. Display your results in a table and then show that the order of accuracy obtained from your results is in agreement with the theory in question **3-1.** [10]

3-3. Establish the error term for the rule: [15]

$$f'''(x) \approx \frac{1}{2h^3}[3f(x+h) - 10f(x) + 12f(x-h) - 6f(x-2h) + f(x-3h)]$$

Problem 4. [19 marks]

4-1. State the second-order Runge-Kutta algorithm (RK2) in terms of it slopes $k1$ and $k2$ (or f_1 and f_2). [6]

4-2 Explain how the Runge-Kutta method can be used to produce a table of the values for the function

$$f(x) = \int_0^x e^{-t^2} dt$$

at 100 equally spaced points in the unit interval. [3]

4-3. Use the procedure explained in **4-2.** and adapt it to compute $f(0.3)$ using RK2 with three iterations, where this time

$$f(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

using RK2 to approximate y(0.3) with 3 steps. [10]

God bless you !!!