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QUALIFICATION : **BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS**	
QUALIFICATION CODE: **07BSAM; 07BSOC**	LEVEL: **7**
COURSE: C**OMPLEX ANALYSIS**	COURSE CODE: **CAN702S**
DATE: **NOVEMBER 2024**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

FIRST OPPORTUNITY EXAMINATION: QUESTION PAPER

EXAMINER: DR. NEGA CHERE

MODERATOR: PROF. FORTUNÉ MASSAMBA

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

ATTACHMENTS:

NONE

This paper consists of 3 pages including this front page.

QUESTION 1 [21]

1.1. Express $\frac{i}{1-i} - \frac{1-i}{i}$ in the form of $x + iy$, where $x, y \in \mathbb{R}$. [6]

1.2. Find the principal argument of the complex number given by $z = \frac{-1}{(1-i)^2}$. [6]

1.3. Compute $\cos\left(\frac{\pi}{3} + i\right)$. [9]

QUESTION 2 [9]

Let $|z + 1| = |z - i|$ be a curve in the complex plane. Then

2.1. describe the type of curve represented by $|z + 1| = |z - i|$. [6]

2.2. determine whether the curve is open or closed, connected or unconnected, bounded or unbounded. Justify your answer. [3]

QUESTION 3 [9]

Find $\cos 4\theta$ and $\sin 4\theta$ in terms of $\sin\theta$ and $\cos\theta$. [9]

QUESTION 4 [25]

Let $f(z) = \begin{cases} \frac{x^3 - y^3 + i(x^3 + y^3)}{x^2 + y^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$. Then show that

4.1. Cauchy-Riemann Equations are satisfied at $(0, 0)$. [15]

4.2. $f'(0, 0)$ does not exist. [10]

QUESTION 5 [14]

Let $f: \mathbb{R}^2 \to \mathbb{R}$ be twice differentiable function.

5.1. What does it mean to say f is harmonic? [2]

5.2. Determine whether $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic or not. If it is harmonic find its harmonic conjugate function $v(x, y)$ such that $f(z) = u(x, y) + iv(x, y)$ is analytic. [12]

QUESTION 6 [22]

Evaluate the following contour integrals.

6.1. $\int_C \left(\frac{z^2-1}{z}\right) dz$ where C is the semicircle parametrized by $z(\theta) = 2e^{i\theta}$, $\pi \leq \theta \leq 2\pi$. [10]

6.2. $\int_C \left(\frac{\cosh z}{z^2+z}\right) dz$ where C is the counter formed by two parts: C_1 defined by the circle $|z + 1| = \frac{1}{2}$ and C_2 defined by the circle $|z| = \frac{1}{2}$, both oriented counterclockwise. [12]

END OF FIRST OPPORTUNITY EXAMINATION QUESTION PAPER