NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

SCHOOL OF AGRICULTURE AND NATURAL RESOURCES SCIENCES

DEPARTMENT OF AGRICULTURAL SCIENCES AND AGRIBUSINESS

QUALIFICATION: BACHELOR OF SCIENCE IN AGRICULTURE	
QUALIFICATION CODE: 07BAGA	**LEVEL: 7**
COURSE CODE: MTA611S	**COURSE NAME**: MATHEMATICS FOR AGRIBUSINESS
SESSION: JUNE 2024	**PAPER:** 1
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
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MODERATOR:	MR TEOFILUS SHIIMI

INSTRUCTIONS

1. Attempt all questions.
2. Write clearly and neatly.
3. Number the answers clearly & correctly.

PERMISSIBLE MATERIALS

1. All written work MUST be done in blue or black ink
2. Calculators allowed
3. The **LAST PAGE** has **FORMULA**
4. No books, notes or other additional aids are allowed

THIS QUESTION PAPER CONSISTS OF 6 PAGE (Including this front page)

QUESTION ONE	[MARKS]
a. Given a cost function $c(q) = (q-3)^{0.5}$, where $c(q)$ is the cost of production in thousands of dollars and q is the quantity of output produced in thousands of units. Use this information to answer the questions below.	
i. Find the $c(12)$.	(3)
ii. Find the domain value that corresponds to a range value of 4.	(3)
iii. Use interval or set notation to express the appropriate domain of the function.	(6)
b. Consider an Agribusiness whose production process is represented by a univariate quadratic function that exhibits a maximum output level and possesses roots at zero and ten units of the input variable. Based on this information, answer the questions below.	
i. Derive the mathematical expression of the Agribusiness's production function.	(3)
ii. Find the range and domain values at the maximum point of the production function.	(5)
iii. Draw and label a graph that illustrates the production function. The graph must clearly show the **roots**, **maxima**, and **y-intercept** points of the production function.	(5)
TOTAL MARKS	**[25]**

QUESTION TWO	[MARKS]
a. Given a multivariate production function $q(l,k) = 10l^{0.25}k^{0.75}$, find the following derivatives q_k, q_{kk} and q_{kl}.	(5)
b. Suppose an Agribusiness's total revenue function, $r(q)$, and cost function, $c(q)$, are represented as: $r(q) = 4000q - 33q^2$; $c(q) = 2q^3 - 3q^2 + 400q + 5000$. Assuming q is the quantity of output, compute the profit-maximizing output level and maximum profit. Furthermore, proves that the profit-maximizing output level is the relative maximum. Show all your calculations.	(8)
c. Using the Difference Quotient, find the first derivation of the function $g(q) = 2q^2$. Show all the steps.	(4)
d. Find the equation of a straight line that is tangent to the function: $f(q) = \ln(4q^2 + 4q - 3)$ at $x = 1$.	(8)
TOTAL MARKS	**[25]**

QUESTION THREE **[MARKS]**

a. For each of the following cost functions, find $c'(q)$:

i. $c(q) = ln(3q^4)$ (3)

ii. $c(q) = \frac{lnq}{q^2}$ (3)

iii. $c(q) = q\,ln(q^2 + 4)$ (4)

b. Suppose a farmer faces the following cost function in his tomato production enterprise:

$$c(k,l) = 3k^3 + 1.5l^2 - 18kl + 17$$

Where $c(k,l)$ is the multivariate production function with labour, l , and capital, k, as the main inputs. Based on this information find: (i) the critical points of the cost function; and (ii) for each critical point, determine if the function is at a **relative maximum, relative minimum, inflection point**, or **saddle point**. (15)

TOTAL MARKS **[25]**

QUESTION FOUR **[MARKS]**

a. Solve the following indefinite integral:

$$\int 3t^2(t^3 - 7)^3\, dt$$ (5)

b. Solve the following definite integral:

$$\int_1^3 \frac{2q}{q^2 + 5} dq$$ (5)

c. Suppose you have observed that the rate at which production in a small stock enterprise changes with respect to improvement in carry capacity is given by:

$$\frac{dy}{dx} = 100\sqrt{x}$$

Where $\frac{dy}{dx}$ is the rate of change with dy being the change in output (i.e., number of animals) and dx being the change in carrying capacity. Furthermore, y and x represent the number of animals and carrying capacity (i.e., number of small stock units (SSU) per hectare), respectively. Derive an equation that represents the output, y, as a function of carrying capacity, x, given initial conditions of 500 animals and carrying capacity of 3 SSU per hectare. (5)

d. Consider an agribusiness that faces a budget constraint of \$108,000 and prices of labour (l) and capital (k) of N\$4 and N\$3 per unit. Suppose the agribusiness's production function is represented by:

$$q(k, l) = k^{0.4}k^{0.5}$$ (10)

Find the units of l and k that maximises the agribusiness's output. Furthermore, compute and interpret the lambda (λ) value.

TOTAL MARKS **[25]**

THE END

FORMULA

Basic Derivative Rules

Constant Rule: $\frac{d}{dx}(c) = 0$

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = cf'(x)$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

Sum Rule: $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

Difference Rule: $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

Derivative Rules for Exponential Functions

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(a^x) = a^x \ln a$

$\frac{d}{dx}(e^{g(x)}) = e^{g(x)} g'(x)$

$\frac{d}{dx}(a^{g(x)}) = \ln(a)\, a^{g(x)}\, g'(x)$

Derivative Rules for Logarithmic Functions

$\frac{d}{dx}(\ln x) = \frac{1}{x}, x > 0$

$\frac{d}{dx}\ln(g(x)) = \frac{g'(x)}{g(x)}$

$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}, x > 0$

$\frac{d}{dx}(\log_a g(x)) = \frac{g'(x)}{g(x) \ln a}$

Basic Integration Rules

1. $\int a\, dx = ax + C$
2. $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{1}{x} dx = \ln|x| + C$
4. $\int e^x\, dx = e^x + C$
5. $\int a^x dx = \frac{a^x}{\ln a} + C$
6. $\int \ln x\, dx = x \ln x - x + C$

Integration by Substitution

The following are the 5 steps for using the integration by substitution metthod:

- Step 1: Choose a new variable ***u***
- Step 2: Determine the value ***dx***
- Step 3: Make the substitution
- Step 4: Integrate resulting integral
- Step 5: Return to the initial variable ***x***

Integration by Parts

The formula for the method of integration by parts is:

$$\int u dv = u \cdot v - \int v du$$

There are three steps how to use this formula:

- Step 1: identify ***u*** and ***dv***
- Step 2: compute ***u*** and ***du***
- Step 3: Use the integration by parts formula

Unconstrained optimization: Univariate functions

The following are the steps for finding a solution to an unconstrained optimization problem:

- Step 1: Find the critical value(s), such that:

  $$f'(a) = 0$$

- Step 2: Evaluate for relative maxima or minima
  - If $f''(a) > 0 \rightarrow$ minima
  - If $f''(a) > 0 \rightarrow$ maxima

Unconstrained optimization: Multivariate functions

The following are the steps for finding a solution to an unconstrained optimization problem:

$f'(a) = 0$	$f''(a) > 0$:	relative minimum at $x = a$
$f'(a) = 0$	$f''(a) < 0$:	relative maximum at $x = a$

Condition	*Minimum*	*Maximum*
FOCs or necessary conditions SOCs or sufficient conditions	$f_1 = f_2 = 0$ $f_{11} > 0$, $f_{22} > 0$, and $f_{11} \cdot f_{22} > (f_{12})^2$	$f_1 = f_2 = 0$ $f_{11} < 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} > (f_{12})^2$
	Inflection point	
	$f_{11} < 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$ or $f_{11} < 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$	
	Saddle point	
	$f_{11} > 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$, or $f_{11} < 0$, $f_{22} > 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$	
	Inconclusive	
	$f_{11} \cdot f_{22} = (f_{12})^2$	

Constrained Optimization

The following are the steps for finding a solution to a constrained optimization problem using the Langrage technique:

- Step 1: Set up the Langrage equation
- Step 2: Derive the First Order Equations
- Step 3: Solve the First Order Equations
- Step 4: Estimate the Langrage Multiplier