NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

SCHOOL OF AGRICULTURE AND NATURAL RESOURCES SCIENCES

DEPARTMENT OF AGRICULTURAL SCIENCES AND AGRIBUSINESS

QUALIFICATION: BACHELOR OF SCIENCE IN AGRICULTURE	
QUALIFICATION CODE: 07BAGA	**LEVEL: 7**
COURSE CODE: MTA611S	**COURSE NAME:** MATHEMATICS FOR AGRIBUSINESS
SESSION: JULY 2024	**PAPER:** 2
DURATION: 3 HOURS	**MARKS:** 100

SECOND OPPORTUNITY/ SUPPLEMENTARY EXAMINATION QUESTION PAPER	
EXAMINER(S)	MR MWALA LUBINDA
MODERATOR:	MR TEOFILUS SHIIMI

INSTRUCTIONS

1. Attempt all questions.
2. Write clearly and neatly.
3. Number the answers clearly & correctly.

PERMISSIBLE MATERIALS

1. All written work MUST be done in blue or black ink
2. Calculators allowed
3. The **LAST PAGE** has **FORMULA**
4. No books, notes or other additional aids are allowed

THIS QUESTION PAPER CONSISTS OF 6 PAGE (Including this front page)

QUESTION ONE	[MARKS]
a. Consider a function $f(a) = \ln(a^2 - 2a - 8)$, compute $f(10)$, and use interval or set notation to express the domain of a function.	(6)
b. Using the limits concept, determine if the function below is continuous at $k = 1$. $$g(k) = \frac{2k - 2}{k^2 - k}$$	(6)
c. Suppose you know that the production function that expresses the relationship between table grapes output (q) and fertilizer application rate (x) is a quadratic function that has: (i) maxima point and (ii) roots at 0 and 50. Based on the provided information, answer the questions below	
i. Derive the mathematical equation of the production function.	(3)
ii. Find the critical point of the production function you have derived in c(i).	(5)
iii. Draw and label a graph that illustrates the production function. The graph must clearly show the roots, maxima, and y-intercept points of the production function.	(5)
TOTAL MARKS	**[25]**

QUESTION TWO	[MARKS]
a. Suppose a small-scale vegetable vendor told you that her original capital value for a vending business is N$25,000. Each week her business income and expenses are N$12,000 and N$7,000, respectively. If all profits are retained in the business, express the value of the business, V, at the end of time, t, weeks as a function of t.	(5)
b. Given a function $h(t) = 4t^2$, use the Difference Quotient to $h'(t)$. Show all the steps.	(4)
c. Find:	
i. $\lim_{h \to 0} \frac{(2+h)^2 - 4}{h}$	(2)
ii. $g''(x)$, given that $g(x) = x^3 - x^2 + 10$.	(3)
iii. $f'(x)$, given that $f(x) = \ln\left(\frac{3x(2x-1)}{5x-2}\right)$.	(5)
d. Find the equation of a straight line that is tangent to the curve: $$g(q) = q^2 - 2q - 24$$ at $q = 2$.	(6)
TOTAL MARKS	**[25]**

QUESTION THREE	[MARKS]

a. Consider the following exponential function:

$$z(x,y) = 3^{7-2x}y^2$$ (10)

Find z_x, z_{yy}, and z_{yx}.

b. Given the following function:

$$z(x,y) = 2y^3 - x^3 + 147x - 54y + 12$$

(15)

Optimize it to: (i) find its critical value(s) and (ii) test whether the function is at relative maximum or minimum.

TOTAL MARKS	[25]

QUESTION FOUR **[MARKS]**

a. Solve: (2)

$$\int \frac{1}{\sqrt[3]{x}} dx$$

b. Solve: (3)

$$\int_0^1 (3x^3 - x + 1)\, dx$$

c. Solve: (5)

$$\int 12y^2(y^3 + 2)dy$$

d. Suppose a Small Diary Enterprise faces fixed costs of N$4000 and marginal costs represented by the function:

$$\frac{dc}{dq} = 250 + 30q + 9q^2$$ (5)

where c is the total cost (in dollars) of producing q kilograms of product. Find the cost of producing 100 kilograms of the product.

e. To meet an order for weaners, a farmer wishes to distribute his production between two farms, farm 1 and farm 2. The total-cost function, $c(q_1, q_2)$, for the weaner production is:

$$c(q_1, q_2) = q_1^2 + 3q_1 + 25q_2 + 1000$$ (10)

where q_1 and q_2 are the numbers of weaners produced at farm 1 and farm 2, respectively. How should the farmer distribute his weaner production across the two farms to minimize production costs? *(Hint: use the Langrage approach with the constraint being: $q_1 + q_2 = 100$).*

TOTAL MARKS **[25]**

THE END

FORMULA

Basic Derivative Rules

Constant Rule: $\frac{d}{dx}(c) = 0$

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = cf'(x)$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

Sum Rule: $\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$

Difference Rule: $\frac{d}{dx}[f(x)-g(x)] = f'(x)-g'(x)$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x)+g(x)f'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

Derivative Rules for Exponential Functions

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(a^x) = a^x \ln a$

$\frac{d}{dx}(e^{g(x)}) = e^{g(x)} g'(x)$

$\frac{d}{dx}(a^{g(x)}) = \ln(a)\, a^{g(x)} g'(x)$

Derivative Rules for Logarithmic Functions

$\frac{d}{dx}(\ln x) = \frac{1}{x}, x > 0$

$\frac{d}{dx}\ln(g(x)) = \frac{g'(x)}{g(x)}$

$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}, x > 0$

$\frac{d}{dx}(\log_a g(x)) = \frac{g'(x)}{g(x)\ln a}$

Basic Integration Rules

1. $\int a\, dx = ax + C$
2. $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{1}{x} dx = \ln|x| + C$
4. $\int e^x\, dx = e^x + C$
5. $\int a^x dx = \frac{a^x}{\ln a} + C$
6. $\int \ln x\, dx = x\ln x - x + C$

Integration by Substitution

The following are the 5 steps for using the integration by substitution metthod:

- Step 1: Choose a new variable *u*
- Step 2: Determine the value *dx*
- Step 3: Make the substitution
- Step 4: Integrate resulting integral
- Step 5: Return to the initial variable *x*

Integration by Parts

The formula for the method of integration by parts is:

$$\int u dv = u \cdot v - \int v du$$

There are three steps how to use this formula:

- Step 1: identify *u* and *dv*
- Step 2: compute *u* and *du*
- Step 3: Use the integration by parts formula

Unconstrained optimization: Univariate functions

The following are the steps for finding a solution to an unconstrained optimization problem:

- Step 1: Find the critical value(s), such that:

  $$f'(a) = 0$$

- Step 2: Evaluate for relative maxima or minima
  - If $f''(a) > 0 \rightarrow$ minima
  - If $f''(a) > 0 \rightarrow$ maxima

Unconstrained optimization: Multivariate functions

The following are the steps for finding a solution to an unconstrained optimization problem:

$f'(a) = 0$	$f''(a) > 0$:	relative minimum at $x = a$
$f'(a) = 0$	$f''(a) < 0$:	relative maximum at $x = a$

Condition	*Minimum*	*Maximum*
FOCs or necessary conditions SOCs or sufficient conditions	$f_1 = f_2 = 0$ $f_{11} > 0$, $f_{22} > 0$, and $f_{11} \cdot f_{22} > (f_{12})^2$	$f_1 = f_2 = 0$ $f_{11} < 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} > (f_{12})^2$
	Inflection point	
	$f_{11} < 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$ or $f_{11} < 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$	
	Saddle point	
	$f_{11} > 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$, or $f_{11} < 0$, $f_{22} > 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$	
	Inconclusive	
	$f_{11} \cdot f_{22} = (f_{12})^2$	

Constrained Optimization

The following are the steps for finding a solution to a constrained optimization problem using the Langrage technique:

- Step 1: Set up the Langrage equation
- Step 2: Derive the First Order Equations
- Step 3: Solve the First Order Equations
- Step 4: Estimate the Langrage Multiplier