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QUALIFICATION : **BACHELOR OF SCIENCE**	
QUALIFICATION CODE: **08BOSC**	LEVEL: **7**
COURSE: **QUANTUM PHYSICS**	COURSE CODE: **QPH702S**
DATE: **NOVEMBER 2024**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

FIRST OPPORTUNITY: QUESTION PAPER

EXAMINER: Professor Dipti Ranjan Sahu
MODERATOR: Professor Vijaya S. Vallabhapurapu

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

This question paper consists of 4 pages including this front page

QUESTION 1: **[20 MARKS]**

1.1 Consider a one-dimensional particle which is confined within the region $0 \leq x \leq a$ and whose wave function is $\psi(x, t) = \sin(\pi x/a) \exp(-i\omega t)$.

1.1.1 Find the potential V(x) (5)

1.1.2 Calculate the probability of finding the particle in the interval $a/4 \leq x \leq 3a/4$ (5)

1.2 List with reason, three properties of a valid wave of a bounded state. (3)

1.3 Explain briefly why photoelectric effect is a quantum phenomenon? (2)

1.4 Compare the energies and wavefunctions of 1-D infinite well and harmonic oscillator (3)

1.5 Why the de-Broglie wave associated with a moving car is not observable? (2)

QUESTION 2: **[20 MARKS]**

2.1. The wave function of the particle is given by

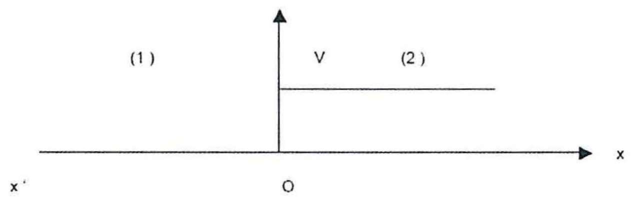
$$\Psi(x) = \sqrt{\frac{2}{a}} \sin(\frac{n\pi}{a} x), \quad x \in [0,a] \quad , n=1, 2, 3....$$

Find the average kinetic energy T (p) of the particle described by this wavefunction (5)

2.2 How to describe a system in quantum mechanics? (2)

2.3 Write the quantum mechanical operators of the classical mechanical expressions K.E. = $\frac{1}{2} mv^2$ in three-dimensional space. (3)

2.4 Consider a potential step of height V as shown in the figure. A particle of energy E > V propagates from $-\infty$ to $+\infty$ (10)



Given A_1 is the amplitude of the incident wave from wave vector K_1 and A_2 be the amplitude of the transmitted wave from wave vector K_2. Find the relation between A_1 and A_2.

QUESTION 3: **[20 MARKS]**

3.1 By applying L^2 operator to the harmonic state $Y_2^0(\theta, \varphi)$, determine the eigenvalue of the state. (10)

3.2 Evaluate the radial eigenfunctions R_{32} of the lithium atom (10)

QUESTION 4: **[20 MARKS]**

4.1 Evaluate the spin matrices Sz for a particle with spin s = $\frac{1}{2}$ (5)

4.2 Evaluate the commutation of L^2 and L_3 and state the consequence of your results. (5)

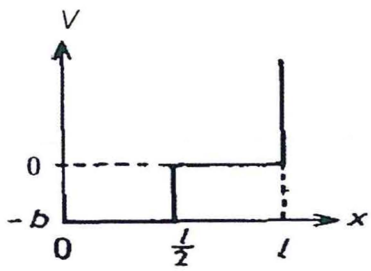
4.3. Evaluate the matrix of L_2 for l = 2. Why is the matrix not diagonal? (10)

QUESTION 5: **[20 MARKS]**

5.1 The perturbation $H' = bx^4$, where b is a constant, is added to the one dimensional harmonic oscillator potential $V(x) = \frac{1}{2} mw^2 x^2$. Determine the correction to the ground state energy to first order in b. The normalized ground state wave function of the one dimensional harmonic oscillator potential is (10)

$$\psi_o = \left(\frac{m\omega}{\hbar\Pi}\right)^{1/4} e^{\frac{-m\omega x^2}{2\hbar}}$$

5.2 A particle moves in a one-dimensional box with a small potential dip (10)



V= ∞ for x< 0 and x>l,
V = -b for 0 < 2 < (l/2) ℓ,
V = 0 for (l/2) ℓ < x < l.

Treat the potential dip as a perturbation to a regular rigid box (V =∞ for x < 0 and x > l, V = 0 for 0 < x < l). Find the first order energy of the ground state. The ground state energy and wavefunction is given by

$$E^{(0)} = \frac{\pi^2\hbar^2}{2ml^2}, \quad \psi^{(0)}(x) = \sqrt{\frac{2}{l}} \sin \frac{\pi x}{l}$$

....

Useful Standard Integral

$$\int_{-\infty}^{\infty} e^{-y^2} dy = \sqrt{\pi}$$

$$\int_{-\infty}^{\infty} y^n e^{-y^2} dy = \frac{\sqrt{\pi}}{n}; \quad n \text{ even}$$

$$0; \quad n \text{ odd}$$

$$\int_{-\infty}^{\infty} e^{-\alpha y^2} e^{-\beta y} dy = \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}} e^{\frac{\beta^2}{4\alpha}}$$

$$\int_0^\infty x^n e^{-x} dx = n!$$

....

Spherical harmonics $\quad Y_l^m(\theta, \varphi) = \left[\frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!}\right]^{\frac{1}{2}} e^{im\varphi} P_l^m(x)$

Associated Legendre polynomials: $P_l^m(x) = (-1)^l \frac{(1-x^2)^{m/2}}{2^l l!} \left(\frac{d}{dx}\right)^{l+m} (1-x^2)^l$, where x = cos θ

Radial eigenfunctions of hydrogen-like atoms:

$$R_{nl}(r) = \left(\frac{2Z}{a_0 n}\right)^{\frac{3}{2}} \left[\frac{(n-l-1)!}{2n[(n+l)!]^3}\right]^{\frac{1}{2}} \left(\frac{2Z}{a_0 n} r\right)^l e^{-\frac{Zr}{a_0 n}} L_{n-l-1}^{2l+1}(\rho)$$, where

$$L_{n-l-1}^{2l+1}(\rho) = \sum_{k=0}^{n-l-1} (-1)^k \cdot \frac{[(n+l)!]^2 \rho^k}{(n-l-1-k)!(2l+1+k)!k!},$$ and $\rho = \frac{2Z}{a_0 n} r$

END OF QUESTION PAPER