P

NAMIBIA UNIVERSITY OF SCIENCE AND TECHNOLOGY

Faculty of Health, Natural Resources and Applied Sciences

School of Natural and Applied Sciences

Department of Mathematics, Statistics and Actuarial Science

13 Jackson Kaujeua Street
Private Bag 13388
Windhoek
NAMIBIA

T: +264 61 207 2913
E: msas@nust.na
W: www.nust.na

QUALIFICATION : **BACHELOR OF SCIENCE IN APPLIED MATHEMATICS AND STATISTICS**	
QUALIFICATION CODE: **07BSAM**	LEVEL: **7**
COURSE: **NUMERICAL METHODS 2**	COURSE CODE: **NUM702S**
DATE: **JANUARY 2025**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **90**

SUPPLEMENTARY/SECOND OPPORTUNITY: QUESTION PAPER

EXAMINER: ***Dr SN NEOSSI-NGUETCHUE***

MODERATOR: ***Prof S.S. MOTSA***

INSTRUCTIONS:

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations. All numerical results must be given using 5 decimals where necessary unless mentioned otherwise.
3. All written work must be done in blue or black ink and sketches must be done in pencil

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

ATTACHEMENTS

None

This paper consists of 3 pages including this front page

Problem 1 [25 Marks]

1-1. Establish the Padé approximation $e^x \approx R_{3,3}(x) = \dfrac{120 + 60x + 12x^2 + x^3}{120 - 60x + 12x^2 - x^3}$ [13]

1-2-1. Find the Fourier sine series for the 2π-periodic function $f(x) = x(\pi - x)$ on $(0, \pi)$. [9]

1-2-2. Use its Fourier representation to find the value of the infinite series. [3]

$$1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \frac{1}{9^3} + \cdots$$

Problem 2 [20 Marks]

For any non negative interger n the Chebyshev polynomial of the first kind of degree n is defined as

$$T_n(x) = \cos\left[n \cos^{-1}(x)\right], \quad \text{for } x \in [-1, 1].$$

2-1. Show the following property [10]

$$< T_m, T_n >= \int_{-1}^{1} \frac{T_m(x)T_n(x)}{\sqrt{1-x^2}}\, dx = \begin{cases} 0, & m \neq n, \\ \pi, & m = n = 0, \\ \frac{\pi}{2}, & m = n \neq 0. \end{cases}$$

2-2. The property in **2-1.** allows us to define the Chebyshev series of $f(x)$ as follows

$$f(x) \sim \sum_{k=0}^{\infty}{}' c_k T_k(x) = \frac{1}{2} c_0 T_0(x) + c_1 T_1(x) + c_2 T_2(x) + \cdots$$

where $c_k =< f, T_k > / < T_k, T_k >$ for $k \geq 1$ and $c_0/2 =< f, T_0 > /\pi$

2-2-1. Determine the Chebyshev series expansion of $f(x) = \sqrt{1 - x^2}$. [10]

Problem 3 [19 Marks]

3-1. Given the integral

$$\int_{0.04}^{1} \frac{1}{\sqrt{x}} dx = 1.6$$

3-1-1. Compute $T(J) = R(J, 0)$ for $J = 0, 1, 2, 3, 4$ using the recursive Trapezoidal rule. [15]

3-1-2. State the two-point and the three-point Gaussian quadrature rules respectively for a continuous function f over the interval $[-1, 1]$. [4]

Problem 4 [26 Marks]

4-1. Assume a 3×3 matrix A is known to have three different real eigenvalues λ_1, λ_2 and λ_3. Assume we know that λ_1 is near -2, λ_2 is near -5 and λ_3 is near -1.

4-1-1. Explain how the power method can be used to find the values of λ_1, λ_2 and λ_3 respectively. [2×3=6]

4-1-2. Discuss how shifting can be used in **4-1-1.** to accelerate the convergence of the power method. [2]

4-2. Use Jacobi's method to find the eigenpairs of the matrix

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

[18]

God bless you !!!