QUALIFICATION : **BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS**	
QUALIFICATION CODE: **07BSAM; 07BSOC**	LEVEL: **7**
COURSE: **COMPLEX ANALYSIS**	COURSE CODE: **CAN702S**
DATE: **JANUARY 2025**	SESSION: **2**
DURATION: **3 HOURS**	MARKS: **100**

SECOND OPPORTUNITY / SUPPLEMENTARY: EXAMINATION QUESTION PAPER

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MODERATOR:	PROF. FORTUN*É* MASSAMBA

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS*:*

1. Non-Programmable Calculator

ATTACHMENTS:

NONE

This paper consists of 3 pages including this front page.

QUESTION 1 [15]

1.1. Find $\left|\left(\sqrt{3} - i\right)^{100}\right|$. [4]

1.2. Determine the principal argument of the complex number $\frac{1-i}{-1-i}$. [6]

1.3. Compute the principal logarithm of the complex number $z = \left(-1 + i\sqrt{3}\right)$. [5]

QUESTION 2 [13]

2.1. Express $f(z) = \frac{1}{i-z}$ in the form of $u(x, y) + iv(x, y)$. [5]

2.2. Find the image of the disk $|z - 1 + i| < 1$ under the transformation

$$w = (1 - i)z + 1 + 2i.$$ [8]

QUESTION 3 [10]

Let $f(z) = \begin{cases} \frac{x^3 - y^3 + i(x^3 + y^3)}{x^2 + y^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$. Then show that $f'(0,0)$ does not exist. [10]

QUESTION 4 [20]

4.1. Use the Cauchy-Riemann conditions to show that $f(z) = e^y \cos x + i\, e^y \sin x$ is nowhere

analytic. [8]

4.2. Determine where $f(z) = x^3 + 3x\, y^2 + i(y^3 + 3\, x^2\, y)$ is differentiable. Is f analytic? Justify

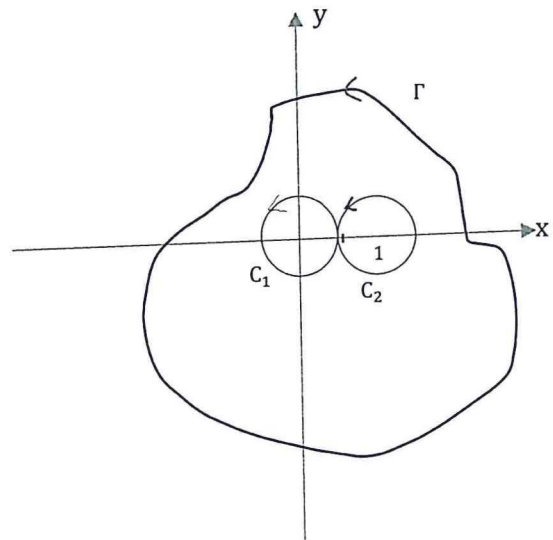your answer. [12]

QUESTION 5 [12]

Let $u(x, y) = 3x^2 + 10xy - 3y^2$. Determine whether $u(x, y)$ is harmonic or not. If it is

harmonic find all analytic function $f(z) = u(x, y) + iv(x, y)$.
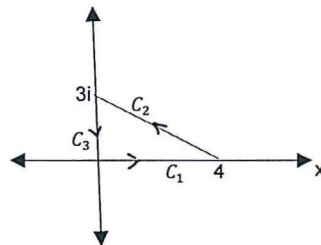
QUESTION 6 [30]

Evaluate the following counter integrals.

6.1. $\int_{\Gamma} \frac{e^z}{(z^2-z)} dz$ where Γ is the counter as shown in the figure below and C_1: $|z| = \frac{1}{2}$ and

C_2: $|z - 1| = \frac{1}{2}$ are interior to Γ oriented counterclockwise. [10]

6.2. $\int_c (e^z + \bar{z}) dz$ where C is the boundary of the triangle with vertices at the points 0, 3i and 4 oriented positively. See the figure below. [20]

END OF SECOND OPPORTUNITY / SUPPLEMENTARY EXAMINATION QUESTION PAPER