NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

SCHOOL OF AGRICULTURE AND NATURAL RESOURCES SCIENCES

DEPARTMENT OF AGRICULTURAL SCIENCES AND AGRIBUSINESS

QUALIFICATION: BACHELOR OF SCIENCE IN AGRICULTURE	
QUALIFICATION CODE: 07BAGA	LEVEL: 7
COURSE CODE: MTA611S	COURSE NAME: MATHEMATICS FOR AGRIBUSINESS
SESSION: JUNE 2025	PAPER: 1
DURATION: 3 HOURS	MARKS: 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
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MODERATOR:	DR TEOFILUS SHIIMI

INSTRUCTIONS
1. Attempt all questions. 2. Write clearly and neatly. 3. Number the answers clearly & correctly.

PERMISSIBLE MATERIALS

1. All written work MUST be done in blue or black ink
2. Calculators allowed
3. The LAST PAGE have FORMULAS

This question paper consists of 6 pages including the front page

QUESTION ONE

a. Consider a function, $f(x) = x^2 - 6x - 7$. Find the domain and the range of the function. (6)

b. Use interval notation to express the domain and range of the following function:

$$g(m) = \frac{2m-1}{m^2-m} \qquad (6)$$

c. Suppose you know that the production function that expresses the relationship between table grapes output (q) and fertilizer application rate (x) is a quadratic function that has: (i) maxima point and (ii) roots at 0 and 75. Based on the provided information, answer the questions below

(i) Derive the mathematical equation of the production function. (5)

(ii) Find the critical point of the production function you have derived in c(i). (5)

(iii) Give the x-intercept and y-intercept points of the production function (3)

TOTAL MARKS [25]

QUESTION TWO

a. Use the Newton's Difference Quotient (or first principle of differentiation) to find the first derivative of the function:

$$h(m) = m^2 - 6m - 7m \qquad (6)$$

To obtain full marks, show all the critical steps in your answer.

b. Find:

(i) $\lim\limits_{m \to 1} \sqrt{\frac{m-1}{m^2+2m-3}}$ (3)

(ii) $\lim\limits_{n \to 6} \frac{\sqrt{n-2}-2}{n-6}$ (5)

(iii) $\lim\limits_{k \to 0} \frac{(2+k)^2-4}{k}$ (2)

c. Find the equation of a straight-line that is tangent to the curve:

$$n = \ln(m^2 - 2m + 24)$$

$$at\ m = 0 \qquad (9)$$

TOTAL MARKS [25]

QUESTION THREE

a. Consider the functions, $f(x) = (3x^4 - 5)^6$ and $g(x) = \log_8 x^4$.
Find:
i. $f'(x)$ (4)
ii. $g'(x)$ (4)

b. Find k_x, k_y and k_{xy}, given the function:

$$k = 3e^{2x}y^2$$ (6)

c. Find the critical points of the function below and test whether it is at a relative maximum, relative minimum, inflection point, or saddle point. Show all calculations.

$$k = 3x^3 - 5y^2 - 225x + 70y + 23$$ (11)

TOTAL MARKS [25]

QUESTION FOUR

a. Find:

i. $\int_0^2 (3x^2 - x - 2)\, dx$ (4)

ii. $\int x^3(12x^2 + 3)\, dx$ (4)

b. Suppose a monopolist served a market that faced the inverse demand function of p=250−2q and a constant marginal cost of production of c=50

What value of $q$maximizes the monopolist's profits?
What is the corresponding price and profit level? (7)

c. To meet an order for weaners, a farmer wishes to distribute his production between two farms, farm 1 and farm 2. The total-cost function, $c(q_1, q_2)$, for the weaner production is:

$$c(q_1, q_2) = q_1^2 + 3q_1 + 25q_2 + 1000$$

where q_1 and q_2 are the numbers of weaners produced at farm 1 and farm 2, respectively. How should the farmer distribute his weaner production across the two farms to minimize production costs? (Hint: use the Langrage approach with the constraint being: $q_1 + q_2 = 100$). (10)

TOTAL MARKS [25]

THE END

FORMULA

Basic Derivative Rules

Constant Rule: $\frac{d}{dx}(c) = 0$

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = cf'(x)$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

Sum Rule: $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

Difference Rule: $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

Derivative Rules for Exponential Functions

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(a^x) = a^x \ln a$

$\frac{d}{dx}(e^{g(x)}) = e^{g(x)} g'(x)$

$\frac{d}{dx}(a^{g(x)}) = \ln(a) a^{g(x)} g'(x)$

Derivative Rules for Logarithmic Functions

$\frac{d}{dx}(\ln x) = \frac{1}{x}, x > 0$

$\frac{d}{dx}\ln(g(x)) = \frac{g'(x)}{g(x)}$

$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}, x > 0$

$\frac{d}{dx}(\log_a g(x)) = \frac{g'(x)}{g(x) \ln a}$

Basic Integration Rules

1. $\int a\,dx = ax + C$
2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{1}{x}\,dx = \ln|x| + C$
4. $\int e^x\,dx = e^x + C$
5. $\int a^x\,dx = \frac{a^x}{\ln a} + C$
6. $\int \ln x\,dx = x\ln x - x + C$

Integration by Substitution

The following are the 5 steps for using the integration by substitution metthod:

- Step 1: Choose a new variable *u*
- Step 2: Determine the value *dx*
- Step 3: Make the substitution
- Step 4: Integrate resulting integral
- Step 5: Return to the initial variable *x*

Integration by Parts

The formula for the method of integration by parts is:

$$\int u\,dv = u \cdot v - \int v\,du$$

There are three steps how to use this formula:

- Step 1: identify *u* and *dv*
- Step 2: compute *u* and *du*
- Step 3: Use the integration by parts formula

Unconstrained optimization: Univariate functions

The following are the steps for finding a solution to an unconstrained optimization problem:

- Step 1: Find the critical value(s), such that:

$$f'(a) = 0$$

- Step 2: Evaluate for relative maxima or minima
  - If $f''(a) > 0 \rightarrow$ minima
  - If $f''(a) > 0 \rightarrow$ maxima

Unconstrained optimization: Multivariate functions

The following are the steps for finding a solution to an unconstrained optimization problem:

$f'(a) = 0$	$f''(a) > 0$:	relative minimum at $x = a$
$f'(a) = 0$	$f''(a) < 0$:	relative maximum at $x = a$

Condition	*Minimum*	*Maximum*
FOCs or necessary conditions SOCs or sufficient conditions	$f_1 = f_2 = 0$ $f_{11} > 0$, $f_{22} > 0$, and $f_{11} \cdot f_{22} > (f_{12})^2$	$f_1 = f_2 = 0$ $f_{11} < 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} > (f_{12})^2$
	Inflection point	
	$f_{11} < 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$ or $f_{11} < 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$	
	Saddle point	
	$f_{11} > 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$, or $f_{11} < 0$, $f_{22} > 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$	
	Inconclusive	
	$f_{11} \cdot f_{22} = (f_{12})^2$	

Constrained Optimization

The following are the steps for finding a solution to a constrained optimization problem using the Langrage technique:

- Step 1: Set up the Langrage equation
- Step 2: Derive the First Order Equations
- Step 3: Solve the First Order Equations
- Step 4: Estimate the Langrage Multiplier