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| QUALIFICATION : **BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS** | |
| QUALIFICATION CODE: **07BSAM** | LEVEL: **6** |
| COURSE: **MATHEMATICAL PROGRAMMING** | COURSE CODE: **MAP602S** |
| DATE: **NOVEMBER 2023** | SESSION: **1** |
| DURATION: **3 HOURS** | MARKS: **100** |

**FIRST OPPORTUNITY EXAMINATION: QUESTION PAPER**

**EXAMINER:** ***Mr. Benson E. Obabueki***

**MODERATOR:** ***Professor Adetayo S. Eegunjobi***

**INSTRUCTIONS**

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.
6. Show all your working/calculation steps.

**PERMISSIBLE MATERIALS:**

1. Non-Programmable Calculator.
2. Metric graph paper to be supplied by the examination department.

**ATTACHEMENTS**

1. None

**This paper consists of 3 pages excluding this front page.**

**Question 1 (8 marks)**

Impact Printing makes two kinds of computer paper using premium or ordinary quality stock. They have a contract to supply at least 5000 cases of paper. There is only enough stock to make 4000 cases of premium paper, but ample stock for ordinary paper. Both kinds are made with the same machine and 1200 hours of machine time are available. Premium paper takes 18 minutes per case and ordinary paper takes 12 minutes per case. The profit on each is \$4/case and \$3/case, respectively.

Model the above statement into a linear programme. You must clearly define your variables unambiguously and name your constraints. DO NOT SOLVE. (8)

**Question 2(13 marks)**

Using a scale of 1cm to 1 unit, solve the following linear program graphically:

$$\begin{aligned} &\textit{Minimize} && T = 10x + 15y \\ &\textit{Subject to} && 12x + 4y \geq 48 \\ & && 4x + 8y \leq 32 \\ & && 2x + 5y \geq 10 \\ & && 0 \leq x < 6 \\ & && y \geq 0 \end{aligned}$$

(13)

**Question 3(26 marks)**

Consider the primal linear program:

$$\begin{aligned} &\textit{Minimize} && T = 32a + 32b \\ &\textit{Subject to} && 4a + 8b \geq 12 \\ & && 8a + 4b \geq 14 \\ & && a, b \geq 0 \end{aligned}$$

3.1 Write down the dual of the linear program. (5)

3.2 Solve the dual of the linear program completely using the simplex method. (12)

3.3 Use the solution of the dual to determine the solution of the primal program. (9)

**Question 4 (18 marks)**

The following linear program is to be solved using the 2-phase method:

$$\begin{aligned} &\textit{Maximize} && Z = 3x + 4y \\ &\textit{Subject to} && 2x + y \leq 600 \\ & && x + y \geq 150 \\ & && 5x + 4y \leq 1000 \\ & && x + 2y \geq 225 \\ & && x, y \geq 0 \end{aligned}$$

4.1 Determine the objective function for phase one. (3)

4.2 Write down the initial tableau for phase one. (2)

4.3 One of the tableaux in the process of phase one is as follows: (5)

| x | y | s1 | s2 | s3 | s4 | A2 | A4 | H | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 0 | 2 | 0 | 0 | 1 | 0 | -1 | 0 | 975 |
| 1 | 0 | 0 | -2 | 0 | 1 | 2 | -1 | 0 | 75 |
| 3 | 0 | 0 | 0 | 1 | 2 | 0 | -2 | 0 | 550 |
| 1 | 2 | 0 | 0 | 0 | -1 | 0 | 1 | 0 | 225 |
| 1 | 0 | 0 | -2 | 0 | 1 | 0 | -3 | 2 | 75 |

Use this tableau to determine the optimal tableau for phase one.

4.4 Express the original function $Z = 3x + 4y$ in terms of the non-basic variables of the final phase one tableau. (3)

4.5 Determine the optimal solution of the given linear program. (6)

**Question 5 (16 marks)**

A brewing company has three plants that produce Soul-Ale. The products are moved from the plants to three warehouses. The costs of moving a crate of 50 bottles from each plant to the different warehouses, the capacities of the plants as well as the demands from the warehouses, are given in the following table:

| | Warehouse 1 | Warehouse 2 | Warehouse 3 | Supply |
|---|---|---|---|---|
| Plant 1 | 9 | 15 | 12 | 10 |
| Plant 2 | 6 | 8 | 13 | 23 |
| Plant 3 | 9 | 3 | 11 | 27 |
| Demand | 21 | 14 | 25 | |

Use the North-west corner method to distribute the products in such a way that the total cost of transportation is minimal. (16)

**Question 6 (17 marks)**

Consider a situation where four jobs (J1, J2, J3, and J4) need to be executed by four workers (W1, W2, W3, and W4), one job per worker. The table below shows the cost of assigning a certain worker to a certain job.

| | J1 | J2 | J3 | J4 |
|---|---|---|---|---|
| W1 | 82 | 83 | 69 | 92 |
| W2 | 77 | 37 | 49 | 92 |
| W3 | 11 | 69 | 5 | 86 |
| W4 | 8 | 9 | 98 | 23 |

Minimize the total cost of the assignment using the Hungarian method. (17)

**End of paper**

**Total marks: 100**