



**NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY**

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QUALIFICATION : BACHELOR of SCIENCE	
QUALIFICATION CODE: 07BOSC	LEVEL: 5
COURSE: PROBABILITY THEORY 1	COURSE CODE: PBT501S
DATE: NOVEMBER 2023	SESSION: 1
DURATION: 3 HOURS	MARKS: 100

FIRST OPPORTUNITY: QUESTION PAPER

EXAMINER: *Dr, Dismas Ntirampeba*

MODERATOR: *Mr, Jonas Amunyela*

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

ATTACHEMENTS

1. Z Table

This paper consists of 4 pages including this front page

Question 1 [20 marks]

- 1.1 Briefly explain the following terminologies as they are applied in set theory and probability theory.
- (i) Set [2]
 - (ii) Random experiment [2]
 - (iii) Sample space [2]
 - (iv) Event [2]
 - (v) Independent events (say A and B) [2]
- 1.2 Consider the events $A = \{1, 3\}$, $B = \{2, 5\}$, $C = \{6\}$, $D = \{3, 4\}$ over a possibility space $S = \{1, 2, 3, 4, 5, 6\}$. With reasons, state whether true or false
- (i) Events A, B and C are pairwise mutually [2]
 - (ii) $\{A, B, C, D\}$ is a partition of the sample space S [3]
 - (iii) $\emptyset^c = \{1, 2, 3, 4, 5\}$ [2]
 - (iv) $A \Delta C = \{1, 3\}$ [3]

Question 2 [20 marks]

- 2.1 Suppose that there were 120 students in the classroom, and that they could be classified as follows:

	Hair color		Total
	Brown	Not Brown	
Gender			
Male	20	-	60
Female	-	30	-
Total	50	-	120

Let $A =$ "student has brown hair" and $B =$ "student is male". Find:

- (i) $P(\bar{A} \cap \bar{B})$ [2]
 - (ii) $P(A \cup \bar{B})$ [2]
 - (iii) $P(\bar{A} | \bar{B})$ [3]
- 2.2 In a certain federal prison, it is known that $2/3$ of the inmates are under 25 years of age. It is also known that $3/5$ of the inmates are male and that $5/8$ of the inmates are female or 25 years of age or older. What is the probability that a prisoner selected at random from this prison is female and at least 25 years old? [5].
- 2.3 A certain federal agency employs three consulting firms (A , B , and C) with probabilities 0.40, 0.35, and 0.25, respectively. From past experience it is known that the probability of cost overruns for the firms are 0.05, 0.03, and 0.15, respectively. Suppose a cost overrun is experienced by the agency.
- (i) What is the probability that the consulting firm involved is company C ? [5]
 - (ii) What is the probability that the consulting firms involved are both A and C companies? [3]

Question 3 [28 marks]

- 3.1 A large industrial firm purchase several new word-processors at the end of each year, the exact number depending on the frequency of repairs in the previous year. Suppose that the number of wordprocessors, X , that are purchased each year has the following probability distribution

x	0	1	2	3
$p(x)$	0.1	0.3	0.4	0.2

- (i) If the cost on new wordprocessors at the end of this year is given by $12000 - 50X^2$, in Namibia dollars, how much can this firm expect to spend on new wordprocessors at the end of this year? [5]
(ii) Find the cumulative distribution of X . [4]
(iii) What is the median value of X ? [1]
- 3.2 A chemist measures the temperature of a solution in $^{\circ}\text{C}$. The measurement is denoted by T , and is normally distributed with mean 30°C and a standard deviation 2°C . The measurement is converted to $^{\circ}\text{F}$ by the equation $F = 1.8T + 32$. Then what is
(i) the mean value of F ? [2]
(ii) the variance of F ? [3]
- 3.3 If a dealers profit, in units of N\$5000, on a new automobile can be looked upon as a random variable X having the density function defined by

$$f(x) = \begin{cases} 2(1-x) & , \text{ if } 0 \leq x \leq 1 \\ 0 & , \text{ otherwise.} \end{cases}$$

- (i) Find the standard deviation of the profit per automobile. [7]
(ii) Find the cumulative distribution of X . [4]
(iii) Use the answer in (ii) to find $P(X \leq \frac{3}{4})$. [2]

Question 4 [32 marks]

- 4.1 The percentage of customers who enter a restaurant and ask to be seated in smoking section is 15%. Suppose that 10 people enter the restaurant.
(i) What is the expected number of people who request a smoking table? [2]
(ii) What the standard deviation of the number of requests for a smoking table? [2]
(iii) What is the probability that at most 2 people request a smoking table? [4]
- 4.2 The number of accidents that occur on assembly line have a Poisson distribution with an average of five accidents per week.
(i) Find the probability that exactly four will occur in a week. [3]
(ii) If the accidents in different weeks are independent of each other, find the expected number of accidents to occur in a month. [2]

- 4.3 Mensa is an organisation whose members possess IQs that are in top 2% of the population. It is known that the IQs are normally distributed with a mean of 100 and a standard deviation of 16.
- (i) Find the proportion of Mensa members who has an IQ greater than 120. [3]
 - (ii) Find the minimum IQ needed to be a Mensa member. [4]
- 4.4 The jurisdiction of a rescue team includes emergencies occurring on a stretch of river that is 4 miles long. Experience has shown that the distance along this stretch, measured in miles from its northernmost point, at which an emergency occurs can be represented by a uniformly distributed random variable over the range 0 to 4 miles. Then, if X denotes the distance (in miles) of an emergency from the northernmost point of this stretch of river,
- (i) find $E(X)$, [2]
 - (ii) find $P(X \geq 2.5)$. [4]
- 4.5 A professor sees students during regular office hours. Time spent with students follows an exponential distribution with a mean of 10 minutes.
- (i) Find the probability that a given student spends fewer than 20 minutes with the professor. [4]
 - (ii) Find the variance of the time spent with students. [2]

END OF QUESTION PAPER

Standard Normal Probabilities

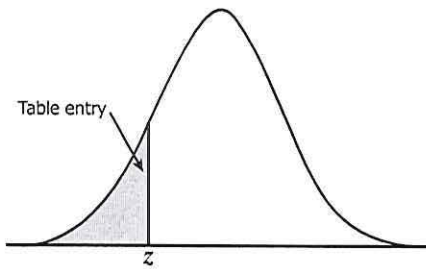


Table entry for z is the area under the standard normal curve to the left of z .

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

