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OF SCIENCE AND TECHNOLOGY

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QUALIFICATION : **Bachelor of Science in Applied Mathematics and Statistics & Bachelor of Science**	
QUALIFICATION CODE: **07BSAM & 07BOS**	LEVEL: **5**
COURSE: **PROBABILITY THEORY 1**	COURSE CODE: **PBT501S**
DATE: **NOVEMBER 2024**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

FIRST OPPORTUNITY: EXAMINATION QUESTION PAPER

EXAMINER: *Dr D Ntirampeba*

MODERATOR: *Mr J Amunyela*

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Write all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

ATTACHEMENTS

1. Z Table

This paper consists of 3 pages excluding this front page

Question 1 [20 marks]

1.1 Briefly explain the following terminologies as they are applied in set theory and probability theory.
(i) Set [2]
(ii) Disjoint sets (say A and B) [2]
(iii) Random experiment [2]
(iv) Sample space [2]
(v) Pairwise mutually exclusive events (say $A_1, A_2, \ldots, A_k$) [2]

1.2 Consider the sample space $S = \{1,2,3,4,5,6,7,8,9,10\}$ and the events $A = \{1,2,5,6\}$, $B = \{3,4,5,6,7\}$, $C = \{2,4,6\}$, $D = \{1,3,6\}$, $E = \{1,2,5,9\}$.
(i) Are A, B, C, D , and E collectively exhaustive? Explain why. [2]
Assuming that all elements are **equiprobable**, find:
(ii) $P(\emptyset - S)$ [2]
(iii) $P(\bar{A} \cup D)$ [2]
(iv) $P(C \Delta D)$ (Hint: Δ means symmetric difference) [4]

Question 2 [30 marks]

2.1 (i) How many distinct permutations can be made from the letters of the word ROWS? [2]
(ii) How many of these permutations start with the letter S followed by letter R? [2]
(iii) State three axioms/rules of probability [3]

2.2 Factory workers are constantly encouraged to practice zero tolerance when it comes to accidents in factories. Accidents can occur because the working environment or conditions themselves are unsafe. On the other hand, accidents can occur due to carelessness or so-called human error. In addition, the worker's shift, 7:00 A.M - 3:00 P.M. (day shift), 3:00 P.M - 11:00 P.M. (evening shift), or 11:00 P.M - 7:00 A.M. (graveyard shift), may be a factor. During the last year, 300 accidents have occurred. The percentages of the accidents for the condition combinations are as follows:

Shift	Unsafe conditions	Human error
Day	5%	32%
Evening	6%	25%
Graveyard	2%	30%

If an accident report is selected randomly from the 300 reports,
(i) what is the probability that the accident occurred on the graveyard shift? [2]
(ii) what is the probability that the accident occurred on either the evening or the graveyard shift? [3]
(iii)If an accident occurred on the evening, what is the probability that it was due to human error? [4]

2.3 A diagnostic test for a disease is said to be 90% accurate in that if a person has the disease, the test will detect it with probability 0.9. Also, if a person does not have the disease, the test will report that he or she does not have it with probability 0.9. Only 1% of the population has the disease in question. If a person is chosen at random from the population and the diagnostic indicates that she has it, what is the probability that she does, in fact, have the disease? [5]

2.4 Three airlines serve a small town in Ohio. Airline A has 50% of the all scheduled flights, Airline B has 30%, and Airline C has the remaining 20%. Their on-time rates are 80%, 65%, and 40%, respectively.
(i) If a plane has just left, what is the probability that it left on time? [3]
(ii) If a plane has just left on time, what is the probability that it was airline A or Airline B [6]

Question 3 [20 marks]

3.1 A company's marketing and accounting departments have determined that if the company markets its newly developed product, the contribution of the product to the firm's profit during the next 6 months will be described by the following:

Profit contribution	Probability
−\$5000	0.2
\$10000	0.3
\$30000	0.5

(i) Find the company's expected profit [3]
(ii) Find the median company's profit (hint: First, find the cumulative distribution of the profit) [5]

3.2 A manufacturing company has developed a machine for cleaning carpet that is fuel-efficient because it delivers carpet cleaner so rapidly. Of interest is a random variable Y , the amount in gallons per minute delivered. It is known that the density function is given by

$$f(y) = \begin{cases} 1 & \text{if } 7 < y < 8 \\ 0 & \text{otherwise} \end{cases}$$

Find:
(i)$E(Y)$ [3]
(ii) $E(Y^2)$ [3]
(iii) $Var(Y)$ [2]
(iv) median of X (hint: First, find $F(x)$). [4]

Question 4 [30 marks]

4.1 A new surgical procedure is successful with a probability $p = 0.8$. Assume the operation is performed ten times and the results are independent of one another. Find:
(i) the expected number of successful operations; [3]
(ii) the probability that fewer than 2 are successful. [4]

4.2 For a certain type of copper wire, it is known that, on the average, 1.5 flaws occur per millimeter. Assuming that the number of flaws is a Poisson random variable,
(i) what is the probability that no flaws occur in a certain portion of wire of length 0.005 meters? [5]
(ii) what is the mean number of flaws in a portion of length 0.005 meters? [3]

4.3 Scores on a Biology examination are assumed to be normally distributed with mean of 78 and a variance 36.
(i) What is the probability that a student taking the examination scores higher than 72? [4]
(ii) What must be the cuttoff point for passing the examination if the examiner wants only the top 28.1% of all scores to be passing? [6]

4.4 A random variable X has a mean $\mu = 10$ and a variance $\sigma^2 = 4$. Use Chebyschev's inequality to estimate $P(|X - 10| \geq 3)$. [5]

END OF QUESTION PAPER

Standard Normal Probabilities

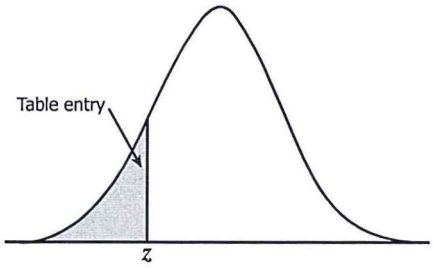

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

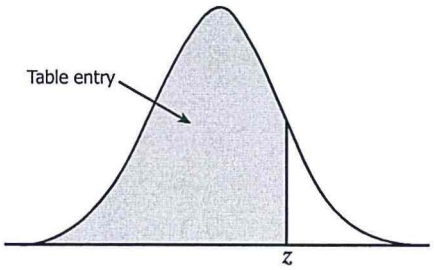

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998