QUALIFICATION: BACHELOR OF SCIENCE IN APPLIED MATHEMATICS AND STATISTICS			
QUALIFICATION CODE:	07BSAM	LEVEL:	5
COURSE:	CALCULUS 1	COURSE CODE:	CLS502S
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DURATION:	3 HOURS	MARKS:	100

FIRST OPPORTUNITY EXAMINATION: QUESTION PAPER

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MODERATOR:	*Dr. Nega Chere*

INSTRUCTIONS (add other relevant instructions):

1. Attempt all the questions in the booklet provided.
2. Please write neatly and legibly using a black or blue inked pen, and sketches must be done in pencil.
3. Do not use the left side margin of the answer script. This must be allowed for the examiner.
4. No books, notes or other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.
6. Show clearly all the steps used in the calculations.

PERMISSIBLE MATERIALS:

1. Non-programmable calculator without a cover.

ATTACHMENTS:

None

This paper consists of 3 pages including this front page

Question 1.

Consider the functions $f(x) = 4x^2 + 9$, $g(x) = \sqrt{1-x^2}$ and $h(x) = 4x^2 - 3$; $x \geq 0$.

1.1 Find the sum of the smallest and the largest numbers in the domain of $\frac{g}{f}$. [5]

1.2 Determine whether g is even, odd or neither. [3]

1.3 Determine whether h^{-1} exists. If it does, find it. [9]

Question 2.

2.1 Find the following limits, if they exist.

a) $\displaystyle\lim_{x \to -3} \frac{4x + 12}{x^3 + 3x^2 - 4x - 12}$. [4]

b) $\displaystyle\lim_{x \to 0^+} (e^x + x)^{\frac{1}{x}}$ [7]

2.2 Use the $\varepsilon - \delta$ method to show that $\displaystyle\lim_{x \to 2}(10x - 6) = 14$. [7]

Question 3.

Let $f(x) = |2x - 10| + 2$.

3.1 Show that f is continuous at $x = 5$. [7]

3.2 Show that f is not differentiable at $x = 5$. [8]

Question 4.

4.1 Use the definition (first principle) to find the derivative of $f(x) = \dfrac{1}{\pi^2} - \dfrac{x-1}{x+\sqrt{2}}$. [7]

4.2 Differentiate each of the following functions

a) $f(x) = (\ln 3)^{\sec x} + \tan^{-1}(\ln 4x)$. [5]

b) $g(x) = (2x^3 + 5)^{\sqrt{x^2+7}}$ [7]

Question 5.

5.1 If the equation $x^2 y + \sin y = 2\pi$ determines a differentiable function f such that $y = f(x)$, find the slope of the tangent line to the graph of this equation at the point $P(1, 2\pi)$. [6]

5.2 Without finding the inverse function f^{-1}, find $(f^{-1})'(b)$, where $f(x) = e^{2x-5}$ and $b = f(-10)$. [7]

Question 6.

Let $f(x) = \dfrac{x^4}{4} - 2x^2 + 4$ and $g(x) = 2x^4 - 8x^3 + 316x - 172$.

6.1 Find the intervals on which f is increasing and on which it is decreasing. [9]

6.2 Find the intervals on which the graph of $y = g(x)$ is concave upwards and on which it is concave downwards. [9]