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QUALIFICATION : **BACHELOR of SCIENCE HONOURS IN APPLIED MATHEMATICS** **BACHELOR of SCIENCE HONOURS IN APPLIED STATISTICS**	
QUALIFICATION CODE: **08BSHM** **08BSHS**	LEVEL: **8**
COURSE: **APPLIED OPERATIONS RESEARCH**	COURSE CODE: **AOR802S**
DATE: **NOVEMBER 2023**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **200** (To be converted to 100%)

FIRST OPPORTUNITY: EXAMINATION QUESTION PAPER

EXAMINER: *Prof Sunday A. Reju*

MODERATOR: *Prof Oluwole D. Makinde*

INSTRUCTIONS

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.
6. Use of COMMA is NOT ALLOWED for a DECIMAL POINT.

PERMISSIBLE MATERIALS

1. Non-Programmable Calculator

ATTACHMENTS

NONE

This paper consists of 4 pages including this front page.

QUESTION 1 [50 MARKS]

(a) A home gardener has a 250-gallon capacity family garden watering tank, initially empty, meant to water the home garden during drought. Consider the following water quantity needed and the prices during probable four levels of drought severity:

DROUGHT SEVERITY	WATER STORAGE NEEDED	WATER PRICES PER GALLON
Mild Drought (MD)	110 Gallons	N$1.00
Average Drought (AD)	180 Gallons	N$1.85
Severe Drought (SD)	230 Gallons	N$2.00
Prolonged Drought (PD)	250 Gallons	N$3.00

Formulate a game model and employ the Minimax criterion technique to determine the gallons of water storage the gardener should have at the current price of N$1 per gallon to avoid wastage and to maximise his saving. (22 Marks)

(b) Consider a competition between two companies, Coca-Cola and Pepsi, and assume the former is thinking of cutting the price of its iconic soda. If it does so, Pepsi may have no choice but to follow suit for its cola to retain its market share. This may result in a significant drop in profits for both companies. Let's assume that the incremental profits that accrue to Coca-Cola and Pepsi are as follows: If both keep prices high, profits for each company increase by $500 million (because of normal growth in demand). If one drops prices (i.e. defects) but the other does not (i.e. cooperates), profits increase by $750 million for the former because of greater market share and are unchanged for the latter. If both companies reduce prices, the increase in soft drink consumption offsets the lower price, and profits for each company increase by $250 million.

(i) Considering the above as an example of applications of Prisoner's dilemma problem, construct the payoff matrix for each company and for the game model, taking Coca-Cola as the row player. (16 Marks)

(ii) What should each company do? (7 Marks)

QUESTION 2 [30 MARKS]

A construction company is bidding for the building of a new College Hostel or its Classroom Block or a combination of both. The construction company must submit a bid proposal, which costs money to prepare, and there are no guarantees that it will be awarded the contract. If the company bids on the Hostel, it has a 35% chance of getting the contract, and it expects to make $162,000 net profit. However, if the company does not get the contract, it loses $11,500. If the company bids on the Classroom Block, there is a 25% chance of getting the contract, and it would net $140,000 in profit. However, if the company does not get the contract, it will lose $5,750.

(a) What should the construction company do? (14 Marks)

(b) How sensitive to the estimate of the probability of the award of a contract is the decision (i):

- in either to build the Hostel or the Classroom Block? (6.5 Marks)
- to the net profit for each case, if awarded the contract? (9.5 Marks)

QUESTION 3 [54 MARKS]

Consider a winning bid of $5.4 million to construct a new plant for a major manufacturer and the manufacturer needs the plant to go into operation within 40 weeks. Below is the list of the various project activities. The third column provides important additional information for coordinating the scheduling of the project crews.

Activity	Activity Description	Immediate Predecessors	Estimated Duration
A	Excavate	—	2 weeks
B	Lay the foundation	A	4 weeks
C	Put up the rough wall	B	10 weeks
D	Put up the roof	C	6 weeks
E	Install the exterior plumbing	C	4 weeks
F	Install the interior plumbing	E	5 weeks
G	Put up the exterior siding	D	7 weeks
H	Do the exterior painting	E, G	9 weeks
I	Do the electrical work	C	7 weeks
J	Put up the wallboard	F, I	8 weeks
K	Install the flooring	J	4 weeks
L	Do the interior painting	J	5 weeks
M	Install the exterior fixtures	H	2 weeks
N	Install the interior fixtures	K, L	6 weeks

(a) Define Critical Path Method (CPM) and Project Evaluation and Review Technique (PERT).

(3 Marks)

(b) Sketch the project network diagram for the above project. (16 Marks)

(c) Distinguish between crashing a project and a project activity. Hence obtain the crash costs per week saved for each activity from the following investigative time-cost trade-off data. (37 Marks)

	Time		Cost			Time		Cost	
Activity	Normal	Crash	Normal	Crash	Activity	Normal	Crash	Normal	Crash
A	2 weeks	1 week	$180,000	$ 280,000	H	9 weeks	6 weeks	$200,000	$ 380,000
B	4 weeks	2 weeks	$320,000	$ 420,000	I	7 weeks	5 weeks	$210,000	$ 270,000
C	10 weeks	7 weeks	$620,000	$ 860,000	J	8 weeks	6 weeks	$430,000	$ 490,000
D	6 weeks	4 weeks	$260,000	$ 340,000	K	4 weeks	3 weeks	$160,000	$ 200,000
E	4 weeks	3 weeks	$410,000	$ 570,000	L	5 weeks	3 weeks	$250,000	$ 350,000
F	5 weeks	3 weeks	$180,000	$ 260,000	M	2 weeks	1 week	$100,000	$ 200,000
G	7 weeks	4 weeks	$900,000	$1,020,000	N	6 weeks	3 weeks	$330,000	$ 510,000

(d) Discuss your observations. (4 Marks)

QUESTION 4 [66 MARKS]

(a) Using the linear programming problem (LPP) approach in obtaining the solution of the game with the following payoff matrix:

$$\begin{bmatrix} 0 & 1 & -1 & 2 \\ -1 & -3 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & -2 & -3 & -1 \end{bmatrix}$$

Obtain the optimal mixed strategies for the two players and the value of the game, discussing your solutions. (50 Marks)

(b) Consider the following Queueing System Data:

Queueing System for 2 Servers with Balking and Reneging

Start Time	Close Time	Balk if queue length is or exceeds	Renege if waiting time exceeds
09:00	09:20	1	2 minutes

Interarrival Time Probability Distribution

Probability	Lower Bound	Upper Bound	Interarrival Time (min)
0.45	0	0.45	1
0.25	0.45	0.7	2
0.1	0.7	0.8	2
0.2	0.8	1	1

Service Time Probability Distribution

Probability	Lower Bound	Upper Bound	Service Time (min)
0.3	0	0.3	2
0.35	0.3	0.65	4
0.35	0.65	1	6

Obtain a Simulation Table for 5 customers using the following header: (16 Marks)

Cust #	Interarrival Time (min)	Arrival Time (hr:min)	Queue Length at Arrival (# cust.)	Balk?	Renege?	Renege Depart (hr:min)
start						
1						
...						

Cust #	Service Time (min)	Server #1 Start (hr:min)	Server #1 End (hr:min)	Server #2 Start (hr:min)	Server #2 End (hr:min)	Renege Wait Time (hr:min)	Wait Time (hr:min)	Total Time (hr:min)
start								
1								
...								

END OF EXAMINATION TOTAL MARKS:200 (CONVERT TO 100%)