Faculty of Health, Natural
Resources and Applied Sciences
School of Natural and Applied Sciences
Department of Mathematics, Statistics and Actuarial Science

| QUALIFICATION : BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS |  |
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| QUALIFICATION CODE: O7BSAM | LEVEL: 7 |
| COURSE: MATHEMATICAL MODELLING 2 | COURSE CODE: MMO702S |
| DATE: JANUARY 2024 | SESSION: 1 |
| DURATION: $\mathbf{3}$ HOURS | MARKS: $\mathbf{2 2 6}$ (To be converted to 100\%) |

## SECOND OPPORTUNITY/SUPPLEMENTARY: EXAMINATION QUESTION PAPER

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EXAMINER:
MODERATOR:
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## Prof Sunday A. Reju

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MODERATOR
Prof Oluwole D. Makinde
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INSTRUCTIONS

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.
6. Use of COMMA is NOT ALLOWED for a DECIMAL POINT.

## PERMISSIBLE MATERIALS

1. Non-Programmable Calculator

## ATTACHMENTS

NONE

This paper consists of 3 pages including this front page.

## QUESTION 1 [40 MARKS]

Let a mass-spring system of mass of 2.5 kg with natural length 0.54 m be stretched to a length of 0.76 m and released by a force of 20.5 N while it is immersed in a fluid of damping constant $c=42$.
(a) Formulate the general model differential equation for the undamped system and find the position of the mass at any time $t$ if it starts from the equilibrium position and is given a push with an initial zero velocity, stating all relevant physical laws. (Your answers correct to 2 decimal places).
(25 Marks)
(b) Formulate the general model differential equation for the damped system, assuming that the damping force is proportional to the velocity of the mass and acts in the direction opposite to the motion. Then obtain only the general solution without using the initial conditions.
(15 Marks)

## QUESTION 2 [31 MARKS]

(a) Consider a local company that produces bowls and mugs and assume that that per unit profit contribution for bowls is given by $\left(\$ 4-0.1 x_{1}\right)$ and that per unit profit contribution for mugs is given by $\left(\$ 5-0.2 x_{2}\right)$.

Formulate and a nonlinear profit maximization problem from the above subject to just a labour constraint given by $x_{1}+2 x_{2}=40$ hours
(15 Marks)
(b) Consider the following data

| x | 1.2 | 1.5 | 2.0 | 2.6 | 3.2 | 4.5 | 5.2 | 5.7 | 6.0 | 6.8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y | 1.1 | 1.3 | 1.6 | 2.0 | 3.4 | 4.1 | 3.2 | 4.5 | 2.5 | 5.2 |

Obtain the normal equations for $f(x)$ defined by (a) above using the above data.
(16 Marks)

## QUESTION 3 [115]

(a) Define the Middle Square Method for generating pseudo-random numbers. Hence using a seed 642246, obtain ten pseudo-random numbers by the method. (26 Marks)

Is there cycling? (YES/NO). If so, when does it occur?
(b) Consider a single server seaport freight system model where seven vessels dock at a seaport to unload cargo according to the following time data (in minutes):

| Vessels | Vessel 1 | Vessel 2 | Vessel 3 | Vessel 4 | Vessel 5 | Vessel 6 | Vessel 7 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Random Inter- <br> Arrival Times | 18 | 55 | 65 | 185 | 212 | 40 | 35 |
| Cargo Unloading <br> Duration | 55 | 45 | 60.5 | 75 | 80 | 70 | 90 |

Find the following performance measures of the Seaport service system (correct to 2 decimal places):
(i) Average wait time.
(ii) Average unload (service) time.
(iii) Average time spent at the seaport.
(iv) Percentage of time the unloading seaport facility is idle.
(c) When do the $3^{\text {rd }}$ and the last vessels leave the seaport?

## QUESTION 4 [40]

(a) A small-scale vocational business firm produces two farming implements: hoes and shovels and realises a net unit profit of $N \$ 125$ per hoe and $N \$ 140$ per shovel. Assume that the firm has up to 250 square metres of iron sheet and 200 metres of wood length to devote to a small farming project plus a signed contract of supplying 10 hoes and 15 shovels to a family farm during the period of the project. Moreover, assume that it requires 2 square metres of iron and 0.65 metre of wood to fabricate a hoe and 3 square metres of iron and 0.85 metre of wood to produce a shovel. Formulate and solve the model for maximising the firm's profits for hoes and shovels.
(20 Marks)
(b) (i) Define post-optimality analysis for linear optimisation problems

Discuss the analysis for change in the firm's profits on hoes, showing all expressions to support your conclusion.

