



NAMIBIA UNIVERSITY  
OF SCIENCE AND TECHNOLOGY

Faculty of Health, Natural  
Resources and Applied  
Sciences

School of Natural and Applied  
Sciences

Department of Mathematics,  
Statistics and Actuarial Science

13 Jackson Kaujeua Street      T: +264 61 207 2913  
Private Bag 13388      E: msas@nust.na  
Windhoek      W: www.nust.na  
NAMIBIA

<b>QUALIFICATION:</b> BACHELOR OF SCIENCE HONOURS IN APPLIED MATHEMATICS AND STATISTICS	
<b>QUALIFICATION CODE:</b>	08BSHS
<b>COURSE :</b>	STATISTICAL QUALITY CONTROL
<b>DATE :</b>	NOVEMBER 2023
<b>DURATION :</b>	3 HOURS
	<b>LEVEL:</b> 8
	<b>COURSE CODE:</b> SQC802S
	<b>SESSION:</b> 1
	<b>MARKS:</b> 100

<b>FIRST OPPORTUNITY EXAMINATION QUESTION PAPER</b>	
<b>EXAMINER</b>	Dr. Jacob Ong'ala
<b>MODERATOR</b>	Dr Isak Neema

#### INSTRUCTION

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

#### PERMISSIBLE MATERIALS

1. Non-Programmable Calculator

#### ATTACHEMENTS

1. Factors for calculation of control charts
2. Normal distribution centered at the target (T)
3. Standard Normal Table

THIS QUESTION PAPER CONSISTS OF 4 PAGES (including the front page)

### **QUESTION ONE - 30 MARKS**

- (a) Select one specific product and one service of your choice, and discuss how the eight dimensions of quality impact its overall acceptance by consumers [8 mks]
- (b) Quality and process improvement occurs most effectively on a project-by-project basis. DMAIC is a structured five-step problem-solving procedure that can be used to successfully complete projects by proceeding through and implementing solutions that are designed to solve root causes of quality and process problems, and to establish best practices to ensure that the solutions are permanent and can be replicated in other relevant business operations. Explain the five steps in DMAIC process (stating clearly the goals and activities in each step) [10 mks]
- (c) State 4 advantages and 2 disadvantages of acceptance sampling [6 mks]
- (d) Describe the following sampling plans
- (i) single-sampling plan [2 mks]
  - (ii) Double-sampling plan [2 mks]
  - (iii) Multiple-sampling plan [2 mks]

### **QUESTION TWO - 25 MARKS**

- (a) Suppose that we have a product (produced under 3-sigma) consisting of an assembly of 100 independent components or parts and all 100 of these parts must be non-defective for the product to function satisfactorily. What is the probability that any specific unit of product is non-defective [3 mks]
- (b) Why would a typical automobile company manufacturing particular vehicles with about 100,000 components each use six-sigma instead of five-sigma for assessing their quality performance? [5 mks]
- (c) Consider the Shewhart  $\bar{x}$  control chart with two-sigma limits
- (i) Find the probability that a single point falls outside the limits when the process is in control [2 mks]
  - (ii) what sample number will produce an out-of control signal [2 mks]
  - (iii) Calculate the standard deviation of the average run length [2 mks]
- (d) A manufacturer of components for automobile transmissions wants to use control charts to monitor a process producing a shaft. The resulting data from 20 samples of 4 shaft diameters that have been measured are:

$$\sum_{i=1}^{20} \bar{x}_i = 10.275$$

and

$$\sum_{i=1}^{20} R_i = 1.012$$

- (i) Find the control limits that should be used on the  $\bar{x}$  and R control charts. [6 mks]
- (ii) Find the probability of detecting a shift of  $0.8\sigma$  in the first sample if the process is monitored under 3 sigma. [3 mks]
- (iii) What is the expected number of samples that must be taken before the shift is detected. [2 mks]

### **QUESTION THREE - 24 MARKS**

A high-voltage power supply should have a nominal output voltage of 350 V. A sample of four units is selected each day and tested for process-control purposes. The data shown in Table 1 give the difference between the observed reading on each unit and the nominal voltage times ten; that is,  $x_i = (\text{observed voltage on unit } i - 350)10$ .

Table 1: Voltage Data for Question 3.

Sample No.	$x_1$	$x_2$	$x_3$	$x_4$
1	6	9	10	15
2	10	4	6	11
3	7	8	10	5
4	8	9	6	13
5	9	10	7	13
6	12	11	10	10
7	16	10	8	9
8	7	5	10	4
9	9	7	8	12
10	15	16	10	13
11	8	12	14	16
12	6	13	9	11
13	16	9	13	15
14	7	13	10	12
15	11	7	10	16
16	15	10	11	14
17	9	8	12	10
18	15	7	10	11
19	8	6	9	12
20	13	14	11	15

- (a) Set up  $\bar{x}$  and R charts on this process. Is the process in statistical control?. [20 mks]
- (b) If specifications are at  $350V \pm 0.5V$ , what can you say about process capability? [4 mks]

### **QUESTION FOUR - 21 MARKS**

Frozen orange juice concentrate is packed in 6-oz cardboard cans. These cans are formed on a machine by spinning them from cardboard stock and attaching a metal bottom panel. By inspection of a can, we may determine whether, when filled, it could possibly leak either on the side seam or around the bottom joint. Such a nonconforming can has an improper seal on either the side seam or the bottom panel. The data is shown in the table below

Sample Size  $n = 50$

Sample Number	Number of Nonconforming Cans, $D_i$	Sample Number	Number of Nonconforming Cans, $D_i$	Sample Number	Number of Nonconforming Cans, $D_i$	Sample Number	Number of Nonconforming Cans, $D_i$
1	12	17	10	31	9	44	6
2	15	18	5	32	6	45	5
3	8	19	13	33	12	46	4
4	10	20	11	34	5	47	8
5	4	21	20	35	6	48	5
6	7	22	18	36	4	49	6
7	16	23	24	37	6	50	7
8	9	24	15	38	3	51	5
9	14	25	9	39	7	52	6
10	10	26	12	40	6	53	3
11	5	27	7	41	2	54	5
12	6	28	13	42	4		
13	17	29	9	43	3		
14	12	30	6				
15	22						
16	8						

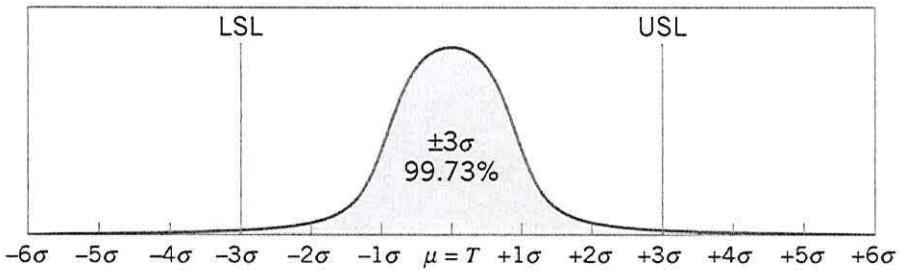
Figure 1: Number of Defective products from each sample of 50

- (a) Set up a control chart to improve the fraction of nonconforming cans produced by this machine using the first 30 samples [10 mks]
- (b) Assuming the out of control points (15 and 23) are due to new batch of raw materials put into production and a relatively inexperienced operator had been temporarily assigned to the machine respectively, Correct the limits of the control charts with this information. [3 mks]
- (c) Using the new set of data, plot the a fraction nonconforming chart using the adjusted limit. [3 mks]
- (d) based on the graph in (c) above perform a statistical test if the process has shifted? [5 mks]

– END OF QUESTIONS –

Observations in Sample, $n$	Chart for Averages						Chart for Standard Deviations						Chart for Ranges					
	Factors for Control Limits			Factors for Center Line			Factors for Control Limits				Factors for Center Line		Factors for Control Limits					
	$A$	$A_2$	$A_3$	$c_4$	$1/c_4$	$B_3$	$B_4$	$B_5$	$B_6$	$d_2$	$1/d_2$	$d_3$	$D_1$	$D_2$	$D_3$	$D_4$		
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0	3.267		
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0	2.574		
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282		
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.114		
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0	2.004		
7	1.134	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076	1.924		
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136	1.864		
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816		
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223	1.777		
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744		
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283	1.717		
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693		
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672		
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347	1.653		
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637		
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622		
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.608		
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.734	1.487	5.891	0.403	1.597		
20	0.671	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	1.549	5.921	0.415	1.585		
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	1.605	5.951	0.425	1.575		
22	0.640	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	1.659	5.979	0.434	1.566		
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	1.710	6.006	0.443	1.557		
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	1.759	6.031	0.451	1.548		
25	0.600	0.153	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	1.806	6.056	0.459	1.541		

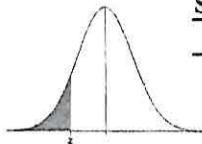
Figure 2: Factors for calculation of control charts



Spec. Limit	Percentage Inside Specs	ppm Defective
±1 Sigma	68.27	317300
±2 Sigma	95.45	45500
±3 Sigma	99.73	2700
±4 Sigma	99.9937	63
±5 Sigma	99.999943	0.57
±6 Sigma	99.99999998	0.002

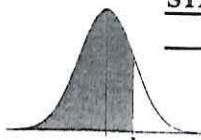
Figure 3: Normal distribution centered at the target (T)

# Standard Normal Distribution Tables



**STANDARD NORMAL DISTRIBUTION:** Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414



**STANDARD NORMAL DISTRIBUTION:** Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997