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QUALIFICATIONS : **BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS AND BACHELOR of SCIENCE**	
QUALIFICATION CODE: **07BSAM ,07BSOC**	LEVEL: **6**
COURSE: **ORDINARY DIFFERENTIAL EQUATIONS**	COURSE CODE: ODE602S
DATE: **JANUARY 2024**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

SECOND OPPORTUNITY/SUPPLEMENTARY: EXAMINATION QUESTION PAPER

EXAMINER: ***Prof Adetayo S. Eegunjobi***

MODERATOR: ***Prof Sunday A. Reju***

INSTRUCTIONS

1. Answer ALL questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left-side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Show all your working /calculation steps.

PERMISSIBLE MATERIALS

1. Non-Programmable Calculator

ATTACHMENTS

1. None

This paper consists of 3 pages including this front page

1. (a) i. Solve the following initial value problem:

$$y'(x) = \frac{y(x) + x}{x}, \quad y(2) = 8, \quad x > 0$$

(3)

ii. Hence or otherwise find $y(x)$ at $x = 8$ (2)

(b) Solve the following initial value problems:

$$y'(x) + \frac{2}{20 - x}y(x) + 1 = 0, \quad y(0) = 20, \quad x \geq 0$$

(5)

(c) i. Suppose a returning student brings the flu virus to his/her boarding house college campus of 5,000 students. Suppose further that the rate at which the virus spreads is proportional not only to the number of infected students but also to the number of students not infected. Determine the number of infected students after 7 days if it is observed that after 5 days the number of infected students is 70. (5)

ii. A tank initially holds 300 liters of liquid, with 20 grams of dissolved salt. Brine, with a salt concentration of 1 gram per liter, is continuously pumped into the tank at a rate of 4 liters per minute. Simultaneously, a well-mixed solution is pumped out of the tank at the same rate. Determine the function $N(t)$, representing the number of grams of salt in the tank at time t. (5)

2. (a) If y_1 and y_2 are two solutions of second order homogeneous differential equation of the form

$$y''(x) + p(x)y'(x) + q(x)y(x) = f(x)$$

where $p(x)$ and $q(x)$ are continuous on an open interval I, derive the formula for $u(x)$ and $v(x)$ by using variation of parameters. (6)

(b) If

$$y_1(x) = 2x + 1, \quad W(y_1, y_2) = 2x^2 + 2x + 1, \quad y_2(0) = 0$$

find $y_2(x)$ (7)

(c) Solve

$$8x^2y''(x) + 16xy'(x) + 2y(x) = 0$$

(7)

3. Solve the following using Laplace Transform

(a)

$$\frac{d^2y(x)}{dt^2} + 2\frac{dy(t)}{dt} + 5y(t) = e^{-t}\sin t, \quad y(0) = 0, \quad y'(0) = 1$$

(7)

(b)

$$\frac{dx(t)}{dt} - y(t) = e^t, \quad \frac{dy(t)}{dt} + x(t) = \sin t, \quad x(0) = 1, \quad y(0) = 0$$

(7)

(c) If $f(t) = t$ and $g(t) = e^{bt}$

i. Find the convolution of $f(t)$ and $g(t)$ (4)

ii. Find the Laplace transform of $f(t) \circledast g(t)$ (2)

4. (a) Solve the Euler equation

$$x^2y''(x) + 15xy'(x) + 58y(x) = 0, \quad y(1) = 1, \quad y'(1) = 0$$

(7)

(b) Solve the following differential equation by method of variation of parameters: $y''(x) + y(x) = \tan x$ (8)

(c) Solve the following differential equation by method of undetermined coefficients:

$$y''(x) + 2y'(x) + 2y(x) = -e^x(5x - 11), y(0) = -1, \quad y'(0) = -3$$

(5)

5. (a) Find at least the first four nonzero terms in a power series expansion about x_0 for the general solution of the following ordinary differential equation at x_0

$$x^2y''(x) - 2xy''(x) + 2y(x) = 0, \quad x_0 = 1$$

(10)

(b) Solve by using Frobenius method:

$$x^2y''(x) + x\left(x - \frac{1}{2}\right)y'(x) + \frac{1}{2}y(x) = 0, \quad \text{at} \quad x_0 = 0$$

(10)

End of Exam!