NAMIBIA UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science honours in Applied Statistics	
QUALIFICATION CODE: 08BSSH	**LEVEL:** 8
COURSE CODE: STP801S	**COURSE NAME:** STOCHASTIC PROCESSES
SESSION: July, 2022	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Prof. RAKESH KUMAR
MODERATOR:	Prof. PETER NJUHO

INSTRUCTIONS
1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Question 1. (Total Marks: 10)

(a) What do you mean by a Martingale. Discuss one example of martingale. (5 Marks)
(b) A particle performs a random walk with absorbing barriers, say 0 and 4. Whenever it is at position r ($0<r<4$), it moves to r+1 with probability p or to r-1 with probability q, p+q=1. But as soon as it reaches 0 or 4, it remains there. The movement of the particle forms a Markov chain. Write the transition probability matrix of this Markov chain. (5 marks)

Question 2. (Total marks: 10)

Classify the stochastic processes according to parameter space and state-space. Give at least two examples of each type. (10 marks)

Question 3. (Total marks: 10)

(a) What is the period of a Markov chain? Differentiate between periodic and aperiodic Markov chains. (5 marks)
(b) What is the nature of state 1 of the Markov chain whose transition probability matrix is given below: (5 marks)

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Question 4. (Total marks: 20)

(a) What is a Poisson process? (5 marks)
(b) Let N(t) be a Poisson process with rate $\lambda > 0$. Prove that the probability of n occurrences by time t is given by

$$P_n(t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}; n = 0,1,2,3, \ldots$$ (15 marks)

Question 5. (Total marks: 20)

(a) Show that the transition probability matrix along with the initial distribution completely specifies the probability distribution of a discrete-time Markov chain. (10 marks)
(b) Suppose that the probability of a dry day (state 0) following a rainy day (state 1) is 1/3 and that probability of a rainy day following a dry day is 1/2. Develop a two-state transition probability matrix of the Markov chain. Given that May 1, 2022 is a dry day, find the probability that May 3, 2022 is a dry day.

(10 marks)

Question 6. (Total marks: 10)

(a) Find the steady-state probabilities of the Markov chain whose one-step transition probability matrix is given below: (7 marks)

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array}$$

(b) State Ergodic theorem. (3 marks)

Q.7 (Total marks: 20)

(a) Derive Kolmogorov backward differential equations. (10 marks)
(b) Derive the steady-state probability distribution of birth-death process. (10 marks)

---------------------------------END OF QUESTION PAPER…………………………………………………………