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OF SCIENCE AND TECHNOLOGY**

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QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSAM	LEVEL: 7
COURSE CODE: AEM702S	COURSE NAME: Applied Econometrics Modelling
SESSION: JANUARY 2025	PAPER: THEORY
DURATION: 3 HOURS	MARKS: 100

SUPPLEMENTARY / SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr D. B. GEMECHU
MODERATOR:	Prof L. PAZVAKAWAMBWA

INSTRUCTIONS
1. There are 6 questions, answer ALL the questions by showing all the necessary steps. 2. Write clearly and neatly. 3. Number the answers clearly. 4. Round your answers to at least four decimal places, if applicable.

PERMISSIBLE MATERIALS

1. Nonprogrammable scientific calculators with no covers.

THIS QUESTION PAPER CONSISTS OF 4 PAGES (Including this front page)

ATTACHMENTS

Four statistical distribution tables (t-, z-, χ^2 - and F-distribution tables)

Question 1 [12 Marks]

- 1.1. List the methodology of Econometrics [4]
- 1.2. State three reasons for inclusion of the random disturbance term in linear regression model [3]
- 1.3. Define autocorrelation and state three causes of autocorrelation of the error term in linear regression model [5]

Question 2 [25 Marks]

- 2.1. Consider the following regression (regression through the origin) model.

$$Y_i = \beta X_i + u_i \text{ for } i = 1, 2, 3, \dots, n$$

where β is the parameter and u_i is the disturbance term.

- 2.1.1. Drive the least square estimator of β [5]

- 2.2. Show that the $Var(\hat{\beta}) = \frac{\sigma^2}{\sum X_i^2}$ [6]

- 2.3. Consider the general (k-variable) linear regression model

$$\begin{matrix} y & = & X & \beta & + & u \\ n \times 1 & & n \times k & k \times 1 & & n \times 1 \end{matrix}$$

If the OLS estimator $\hat{\beta} = (X'X)^{-1}X'y$ is assumed to be unbiased estimator of β , derive the variance covariance matrix of $\hat{\beta}$ [8]

- 2.4. Consider the following model

$$Y_i = \beta_1 + \beta_2 X_i + u_i,$$

where $E(u_i^2) = \sigma^2 X_i^2$.

- 2.4.1. What assumption of the classical linear regression model is violated in this model? [2]
- 2.4.2. Perform an appropriate transformation of the equation to remedy the problem. You are also expected to show that the problem has been solved after appropriate remedial transformation, thus, compute the error variance of the transformed model and show that it is no longer dependent of X_i^2 . [4]

Question 3 [12 Marks]

3. Consider the following regression result for expenditure on new plant and equipment (Y) on sales (X) in billions of dollars and lagged value of Y.

$$\begin{aligned} \hat{Y}_t &= -15.104 + 0.629X_t + 0.272Y_{t-1} \\ se &= (4.7294) \quad (0.0978) \quad (0.1148) \\ d &= 1.5185, \quad \text{durbin } h = 1.3403 \end{aligned}$$

Answer the following questions based on this result.

- 3.1. Assuming that

$$Y_t = \alpha + \beta X_t^* + u_t,$$

where X_t^* are the desired sales and Y = expenditure on new plant and equipment. Derive the adjusted expectation model. Hint: use the adjusted expectation hypothesis: $X_t^* - X_{t-1}^* = \gamma (X_t - X_{t-1})$ [6]

- 3.2. Compute the coefficient of expectation [2]
- 3.3. Estimate the parameters of this model [4]

Question 4 [20 Marks]

4. Consumption expenditure is the main component of the Gross National Product. There are a number of factors that could determine consumption expenditure. Household Income and Expenditure Surveys (HIES) provide data required to assess trends in economic well-being. In Namibia, household expenditure data is collected by the Namibia Statistics Agency (NSA). Suppose a researcher employed a multiple linear regression model to study factors [income (income), and household size (hhsz)] that could determine the total household consumption expenditure (cons_exp) in Namibia using the 2015/2016 NHIES data (*modified for this problem*). Answer the following questions based on the summary of sample values and the statistical software (R-package) output provided below.

Summary of sample values

n=1396

Call:

lm(formula = cons_exp ~ income + hhsz)

Residuals:

Min	1Q	Median	3Q	Max
-65257	-2203	468	3628	12896

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	2935.3017	_____ (4.1.1)	5.5596	< 0.001
income	0.8216	_____ (4.1.2)	112.639	< 0.001
hhsz	517.0920	_____ (4.1.3)	5.8556	< 0.001

Residual standard error: 7150 on 1394 degrees of freedom

Multiple R-squared: 0.9013, Adjusted R-squared: _____ (4.1.4)

F-statistic: 6362 on 2 and 1394 DF, p-value: < 0.001

Variance covariance matrix of the regression coefficient

	(Intercept)	income	hhsz
(Intercept)	278748.179	-2.816	-26896.792
income	-2.816	0.00005329	-0.001
hhsz	-26896.792	-0.001	7798.283

- 4.1. Complete the above results 4.1.1-4.1.4. [5]
- 4.2. Interpret the coefficients of the model. [2]
- 4.3. At 5% level of significance, test for the overall significance of the model. You are expected to state the null and alternative hypothesis, the test statistics, decision and conclusions. [3]
- 4.4. At 5% level of significance, test for significance of individual partial coefficients of the model. You are expected to state the null and alternative hypothesis, the test statistics, decision and conclusions. [3]
- 4.5. Compute an unbiased estimate of the residual variance. [2]
- 4.6. Interpret the coefficient of multiple determination. [2]
- 4.7. Compute the predicted consumption expenditure for a house with income N\$10,000 and household size of 3. [3]

Question 5 [10 Marks]

5. Consider a distributed-lag model for expenditure on plant and equipment (Y) which is assumed to depend on sales (X) in the current year and in the preceding 2 years. Thus,

$$Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + u_t$$

- 5.1. If the β_i 's can be approximated with a second-degree polynomial, then show that the Almon polynomial lag model for Y_t is

$$Y_t = \alpha + a_0 Z_{0t} + a_1 Z_{1t} + a_2 Z_{2t} + u_t,$$

$$\text{where } Z_{0t} = \sum_{i=0}^2 X_{t-i}, Z_{1t} = \sum_{i=0}^2 i X_{t-i} \text{ and } Z_{2t} = \sum_{i=0}^2 i^2 X_{t-i} \quad [6]$$

- 5.2. If the fitted Almon polynomial lag model is

$$\hat{Y}_t = -23.0817 - 0.5197z_{0t} + 2.4843z_{1t} - 1.0080z_{2t}$$

$$se = (4.6543) \quad (0.2126) \quad (0.9130) \quad (0.4461),$$

Estimate the parameters (β 's) of the distributed lag model and state the final fitted distributed lag model. [4]

Question 6 [21 Marks]

6. Consider the following hypothetical structural model.

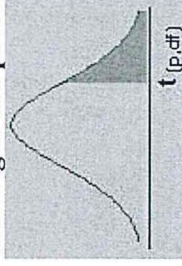
$$I_t = \alpha_0 + \alpha_1 Y_t + u_t$$

$$Y_t = Q_t + I_t$$

- 6.1. Show that Y_t and u_t are correlated. Assume that u_t satisfies the assumptions of the classical linear regression model. Hint: Just show that $cov(Y_t, u_t) \neq 0$ [8]
6.2. Write the reduced form equation expressed in the form of Y_t and I_t . Determine which of the preceding equations are identified (either just or over) [10]
6.3. Describe the steps involved in method of indirect least squares (ILS) used to obtain the estimates of the structural coefficients [3]

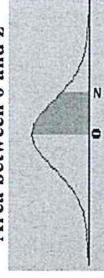
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t table with right tail probabilities



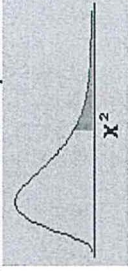
df\p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460

Area between 0 and z



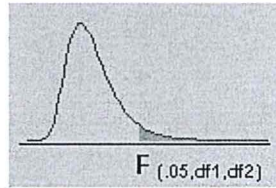
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Right tail areas for the *Chi-square* Distribution



df\area	.995	.990	.975	.950	.900	.750	.500	.250	.100	.050	.025	.010	.005
1	0.00004	0.00016	0.00098	0.00393	0.01579	0.10153	0.45494	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
2	0.01003	0.02010	0.05064	0.10259	0.21072	0.57536	1.38629	2.77259	4.60517	5.99146	7.37776	9.21034	10.59663
3	0.07172	0.11483	0.21580	0.35185	0.58437	1.21253	2.36597	4.10834	6.25139	7.81473	9.34840	11.34487	12.83816
4	0.20699	0.29711	0.48442	0.71072	1.06362	1.92256	3.35669	5.38527	7.77944	9.48773	11.14329	13.27670	14.86036
5	0.41174	0.55430	0.83121	1.14548	1.61031	2.67460	4.35146	6.62568	9.23636	11.07050	12.83250	15.08627	16.74960
6	0.67573	0.87209	1.23734	1.63538	2.20413	3.45460	5.34812	7.84080	10.64464	12.59159	14.44938	16.81189	18.54758
7	0.98926	1.23904	1.68987	2.16735	2.83311	4.25485	6.34581	9.03715	12.01704	14.06714	16.01276	18.47531	20.27774
8	1.34441	1.64650	2.17973	2.73264	3.48954	5.07064	7.34412	10.21885	13.36157	15.50731	17.53455	20.09024	21.95495
9	1.73493	2.08790	2.70039	3.32511	4.16816	5.89883	8.34283	11.38875	14.68366	16.91898	19.02277	21.66599	23.58935
10	2.15586	2.55821	3.24697	3.94030	4.86518	6.73720	9.34182	12.54886	15.98718	18.30704	20.48318	23.20925	25.18818
11	2.60322	3.05348	3.81575	4.57481	5.57778	7.58414	10.34100	13.70069	17.27501	19.67514	21.92005	24.72497	26.75685
12	3.07382	3.57057	4.40379	5.22603	6.30380	8.43842	11.34032	14.84540	18.54935	21.02607	23.33666	26.21697	28.29952
13	3.56503	4.10692	5.00875	5.89186	7.04150	9.29907	12.33976	15.98391	19.81193	22.36203	24.73560	27.68825	29.81947
14	4.07467	4.66043	5.62873	6.57063	7.78953	10.16531	13.33927	17.11693	21.06414	23.68479	26.11895	29.14124	31.31935
15	4.60092	5.22935	6.26214	7.26094	8.54676	11.03654	14.33886	18.24509	22.30713	24.99579	27.48839	30.57791	32.80132
16	5.14221	5.81221	6.90766	7.96165	9.31224	11.91222	15.33850	19.36886	23.54183	26.29623	28.84535	31.99993	34.26719
17	5.69722	6.40776	7.56419	8.67176	10.08519	12.79193	16.33818	20.48868	24.76904	27.58711	30.19101	33.40866	35.71847
18	6.26480	7.01491	8.23075	9.39046	10.86494	13.67529	17.33790	21.60489	25.98942	28.86930	31.52638	34.80531	37.15645
19	6.84397	7.63273	8.90652	10.11701	11.65091	14.56200	18.33765	22.71781	27.20357	30.14353	32.85233	36.19087	38.58226
20	7.43384	8.26040	9.59078	10.85081	12.44261	15.45177	19.33743	23.82769	28.41198	31.41043	34.16961	37.56623	39.99685
21	8.03365	8.89720	10.28290	11.59131	13.23960	16.34438	20.33723	24.93478	29.61509	32.67057	35.47888	38.93217	41.40106
22	8.64272	9.54249	10.98232	12.33801	14.04149	17.23962	21.33704	26.03927	30.81328	33.92444	36.78071	40.28936	42.79565
23	9.26042	10.19572	11.68855	13.09051	14.84796	18.13730	22.33688	27.14134	32.00690	35.17246	38.07563	41.63840	44.18128
24	9.88623	10.85636	12.40115	13.84843	15.65868	19.03725	23.33673	28.24115	33.19624	36.41503	39.36408	42.97982	45.55851
25	10.51965	11.52398	13.11972	14.61141	16.47341	19.93934	24.33659	29.33885	34.38159	37.65248	40.64647	44.31410	46.92789
26	11.16024	12.19815	13.84390	15.37916	17.29188	20.84343	25.33646	30.43457	35.56317	38.88514	41.92317	45.64168	48.28988
27	11.80759	12.87850	14.57338	16.15140	18.11390	21.74940	26.33634	31.52841	36.74122	40.11327	43.19451	46.96294	49.64492
28	12.46134	13.56471	15.30786	16.92788	18.93924	22.65716	27.33623	32.62049	37.91592	41.33714	44.46079	48.27824	50.99338
29	13.12115	14.25645	16.04707	17.70837	19.76774	23.56659	28.33613	33.71091	39.08747	42.55697	45.72229	49.58788	52.33562
30	13.78672	14.95346	16.79077	18.49266	20.59923	24.47761	29.33603	34.79974	40.25602	43.77297	46.97924	50.89218	53.67196

Table for $\alpha=.05$



		df1										
df2\df1		1	2	3	4	5	6	7	8	9	10	12
1		161.448	199.500	215.707	224.583	230.162	233.986	236.768	238.883	240.543	241.882	243.906
2		18.513	19.000	19.164	19.247	19.296	19.329	19.353	19.371	19.384	19.396	19.413
3		10.128	9.552	9.277	9.117	9.014	8.941	8.887	8.845	8.812	8.786	8.745
4		7.709	6.944	6.591	6.388	6.256	6.163	6.0942	6.041	5.998	5.964	5.912
5		6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735	4.678
6		5.987	5.143	4.757	4.533	4.387	4.284	4.207	4.147	4.099	4.060	3.999
7		5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.676	3.637	3.575
8		5.318	4.459	4.066	3.838	3.688	3.581	3.501	3.438	3.388	3.347	3.284
df2	9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.229	3.178	3.137	3.073
10		4.965	4.103	3.708	3.478	3.326	3.217	3.136	3.072	3.020	2.978	2.913
11		4.844	3.982	3.587	3.358	3.204	3.095	3.012	2.948	2.896	2.854	2.788
12		4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.687
13		4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.604
14		4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.645	2.602	2.534
15		4.543	3.682	3.287	3.056	2.901	2.791	2.707	2.641	2.587	2.544	2.475
16		4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.537	2.494	2.425
17		4.451	3.591	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450	2.381
18		4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412	2.342
19		4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378	2.308
20		4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.441	2.393	2.348	2.278