Faculty of Health, Natural Resources and Applied Sciences

School of Natural and Applied Sciences

Department of Mathematics, Statistics and Actuarial Science

QUALIFICATION : Bachelor of Science in Applied Mathematics and Statistics & Bachelor of Science	
QUALIFICATION CODE: 07BSAM & 07BOS	LEVEL: **5**
COURSE: **PROBABILITY THEORY 1**	COURSE CODE: **PBT501S**
DATE: **JANUARY 2025**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

SECOND OPPORTUNITY / SUPPLEMENTARY: EXAMINATION QUESTION PAPER

EXAMINER:	*Dr D Ntirampeba*
MODERATOR:	*Mr J Amunyela*

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Write all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

ATTACHEMENTS

1. Z Table

This paper consists of 3 pages excluding this front page

Question 1 [20 marks]

1.1 Briefly explain the following terminologies as they are applied in set theory and probability theory.
(i) Set [2]
(ii) Random experiment [2]
(iii) Sample space [2]
(iv) Event [2]
(v) Independent events (say A and B) [2]

1.2 Consider the sample space $S = \{coper, sodium, nitrogen, potassium, uranium, oxygen, zinc\}$ and the all elements are **equiprobable**, and further consider the following events:
$A = \{copper, sodium, zinc\}$, $B = \{sodium, nitrogen, potassium\}$,
$C = \{oxygen\}$.

1.2.1 With reasons, state whether true or false that $\{A, B, C\}$ is a partition of S. [2]

1.2.2 Find the following probabilities:
(i) $P(\bar{A})$ [1]
(ii) $P(A \cup B)$ [2]
(iii) $P(B - A)$ [2]
(iv) $P(B \oplus C)$ (hint: $\oplus$ means symmetric difference) [3]

Question 2 [25 marks]

2.1 To determine whether drinking alcoholic beverages has an effect on the bacteria that cause ulcers, researchers developed the following table of the joint probabilities.

Number of alcoholic Drink per Day	Ulcer	No Ulcer
None	0.01	0.22
One	0.03	0.19
Two	0.03	0.32
More than two	0.04	0.16

(i) What is the proportion of people have ulcers? [2]
(ii) What is the probability that someone has an ulcer or does not drink alcoholic beverages? [3]
(iii) If someone has an ulcer, what is the probability that he or she drinks alcoholic beverages? [5]

2.2 A rare disease exists with which only 1 in 500 is affected. A test for the disease exists, but of course it is not infallible. A correct positive result (patient actually has the disease) occurs 95% of the time, while a false positive result (patient does not have the disease) occurs 1% of the time. If a randomly selected individual is tested and the result is positive, what is the probability that the individual has the disease? [5]

1

2.3 A certain federal agency employs three consulting firms (A, B, and C) with probabilities 0.40, 0.35, and 0.25, respectively. From past experience it is known that the probability of cost overruns for the firms are 0.05, 0.03, and 0.15, respectively. Suppose a cost overrun is experienced by the agency.

(i) What is the probability that the consulting firm involved is company C? [6]

(ii) What is the probability that the consulting firms involved is company A and B? [4]

Question 3 [30 marks]

3.1 The natural remedy echinacea is reputed to boost the immune system, which will reduce the number of flu and colds. A six-month study was undertaken to determine whether the remedy works. From the study, the following probability distribution of the number of respiratory infections per year (X) for echinacea users was produced.

x	0	1	2	3	4
$p(x)$	0.45	0.31	0.17	0.06	0.01

(i) Find is the expected number of respiratory infections among echinacea users? [3]

(ii) Find the standard deviation of the expected number of respiratory infections among echinacea users. [5]

(iii) Find the cumulative distribution of X. [5]

(iv) What is the median value of X? [1]

3.2 A chemist measures the temperature of a solution in ^{0}C. The measurement is denoted by T, and is normally distributed with mean 30^0C and a standard deviation 2^0C. The measurement is converted to ^{0}F by the equation $F = 1.8T + 32$. Then what is

(i) the mean value of F? [2]

(ii) the median value of F? [2]

(iii) the variance of F? [2]

3.3 A cereal manufacturer is aware that the weight of the product in the box varies slightly from box to box. In fact, considerable historical data have allowed the determination of the density function that describes the probability structure for the weight (in ounces). Letting X be the random variable weight, in ounces, the density function can be described as

$$f(y) = \begin{cases} \frac{2}{5} & \text{if} \quad 23.75 < y < 26.25 \\ 0 & \text{otherwise} \end{cases}$$

Find:

(i) $E(Y)$ [3]

(ii) $E(Y^2)$ [3]

(iii) $Var(Y)$ [2]

(iv) median of y (hint: First, find $F(Y)$). [4]

Question 4 [25 marks]

4.1 A certain type of tomato seed germinates 90% of time. A backyard farmer planted 10 seeds. Find
   (i) the probability that at most 3 seeds germinates,   [4]
   (ii) the mean number of seeds that will germinate.   [2]

4.2 For a certain type of copper wire, it is known that, on the average, 1.5 flaws occur per millimeter. Assuming that the number of flaws is a Poisson random variable,
   (i) what is the probability that no flaws occur in a certain portion of wire of length 5 millimeters?   [3]
   (ii) what is the mean number of flaws in a portion of length 5 millimeters?   [2]

4.3 The width of bolts of fabric is normally distributed with mean 950 mm and standard deviation 10 mm.
   (i) What probability that a randomly chosen bolt has a width between 947 and 958 mm?[4]
   (ii) What is the appropriate value for c such that a randomly chosen bolt has a width less than c with probability 0.8531?   [5]

4.4 A random variable X has a mean $\mu = 10$ and a variance $\sigma^2 = 4$. Use Chebyshevs theorem to find the value of the constant c such that $P(|X - 10| \geq c) \leq 0.04$ [5]

END OF QUESTION PAPER
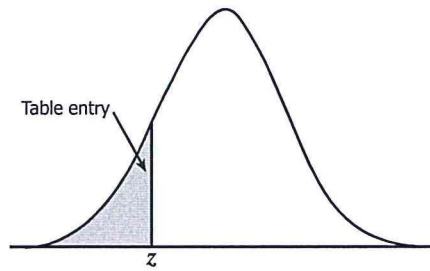
Standard Normal Probabilities

Table entry

z

Table entry for z is the area under the standard normal curve to the left of z.

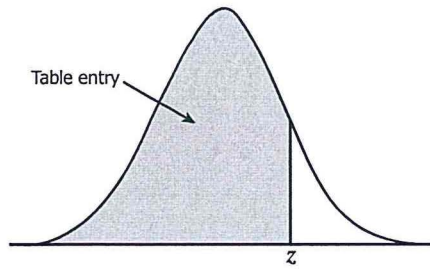| z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998