QUALIFICATION : BACHELOR OF SCIENCE IN APPLIED MATHEMATICS AND STATISTICS	
QUALIFICATION CODE: **07BSAM; 07BSOC**	LEVEL: **7**
COURSE: **COMPLEX ANALYSIS**	COURSE CODE: **CAN702S**
DATE: **NOVEMBER 2023**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

FIRST OPPORTUNITY EXAMINATION: QUESTION PAPER

EXAMINER:	DR. NEGA CHERE
MODERATOR:	PROF. FORTUNÉ MASSAMBA

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly with black or blue ink pen.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

ATTACHMENTS:

NONE

This paper consists of 2 pages including this front page.

1. (a) Find the real and imaginary part of $\frac{z-2}{1-z}$. (6)

    (b) Find the image of the disk $\mid z+1 \mid< 2$ under the transformation
        $w = (1 + 2i)z + 2 - i$. (5)

2. Evaluate $\int_{1-2i}^{1+2i}(e^{\pi z} + z + i)\,dz$. (8)

3. Let $f(z) = \begin{cases} \frac{(\bar{z})^2}{z} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$, where $z = x + iy$. Then show that

    (a) $f(z)$ is not analytic at $(0, 0)$. (13)

    (b) the Cauchy-Riemann Equations are satisfied at $(0, 0)$. (13)

4. Show that $u(x, y) = y^3 - 4xy - 3x^2y$ is harmonic and find its harmonic conjugate $v(x, y)$
    for which $f(z) = u(x, y) + i\,v(x, y)$ is analytic. (15)

5. Evaluate $\int_C (y - x - ix^2)dz$ where C is the counter joining 0 to $1 + i$, $1 + i$ to i and i to
    $-1 + 2i$. (17)

6. Without evaluating the integral show that $\mid \int_C \frac{z+5}{z^3-3}\,dz \mid \leq \pi$, where C is the semicirle
    with center the origin and radius 3, oriented positively. (8)

7. (a) Evaluate $\int_C \frac{e^{z^2}}{z^2+4}\,dz$ where C is the circle $\mid z - \frac{1}{2}i \mid= 1$. (4)

    (b) Evaluate $\int_C \frac{e^{\pi z}}{z^2+9}\,dz$ where $C = C_1 + C_2$ and $C_1 = \{\mid z + 3i \mid= 3\}$,
        $C_2 = \{\mid z - 3i \mid= 3\}$. (11)

END OF FIRST OPPORTUNITY EXAMINATION QUESTION PAPER