Faculty of Health, Natural
Resources and Applied
Sciences
School of Natural and Applied
Sciences
Department of Mathematics,
Statistics and Actuarial Science

| QUALIFICATION:BACHELOR of SCIENCE HONOURS IN APPLIED MATHEMATICS <br> BACHELOR of SCIENCE HONOURS IN APPLIED STATISTICS <br> QUALIFICATION CODE: 08BSHM <br> 08BSHS <br> COURSE: APPLIED OPERATIONS RESEARCH LEVEL: 8 |  |
| :--- | :--- |
| DATE: JANUARY 2024 | COURSE CODE: AOR802S |
| DURATION: $\mathbf{3}$ HOURS | MARKS: 161 (To be converted to 100\%) |

SECOND OPPORTUNITY/SUPPLEMENTARY: EXAMINATION QUESTION PAPER

EXAMINER:
MODERATOR:

Prof Sunday A. Reju
Prof Oluwole D. Makinde

INSTRUCTIONS

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.
6. Use of COMMA is NOT ALLOWED for a DECIMAL POINT.

PERMISSIBLE MATERIALS

1. Non-Programmable Calculator

## ATTACHMENTS

NONE

This paper consists of 3 pages including this front page.

## QUESTION 1 [20 MARKS]

(a) Discuss the Kendall's classification of Queuing Systems?
(5 Marks)
Explain specifically the $M / M / 1$ queuing system and the process $N(t)$ describing its state at time $t$ as a birth-death process. Provide its state independent parameter equations and define its Traffic Intensity.
(b) Consider a drive-in banking service modelled as an $M / M / 1$ queuing system with customer arrival rate of 2 per minute. It is desired to have fewer than 5 customers line up $99 \%$ of the time. How fast should the service rate be?
(4 Marks)
(c) Trucks arrive at garage for a stop-over service according to a Poisson process at a rate of one per every 13 minutes, and the garage service time is an exponential rate variable with mean 9 minutes.
(i) Find the average number $L$ of trucks, the average time $W$ a truck spends in the garage, and the average time $W_{q}$ a truck spends in waiting for service.
(5 Marks)
(ii) Due to increased traffic, suppose that the arrival rate of the trucks increases by $5 \%$. Find the corresponding changes in $\mathrm{L}, \mathrm{W}$, and $W_{q}$.
(iii) Discuss your observations.
(1 Marks)

## QUESTION 2 [50 MARKS]

(a) Discuss Game Reduction by Dominance procedure.
(4 Marks)
(b) Simplify by using reduction by dominance the game defined by the following payoff matrix, showing progressively the reduced pay-off matrix:
(18 Marks)

> B
> $a \quad b \quad c$
> $\left.\begin{array}{cc} & A \\ A & B\end{array} \begin{array}{ccc}2 & -1 & -5 \\ 4 & -4 & 2 \\ 3 & -3 & -8 \\ 2 & -5 & -4\end{array}\right]$
(c) Distinguish between game players' pure and mixed strategies, with clear characteristics of the latter.
(6 Marks)
(d) Eufenia has a 250 -gallon capacity home heating oil tank, presently empty, meant to store oil against the next winter. Consider the following winter heating oil quantity needed and the oil prices during probable four levels of winter severity:

| WINTER SEVERITY | OIL STORAGE NEEDED | OIL PRICES PER GALLON |
| :--- | :---: | :---: |
| Mild Winter (MW) | 110 Gallons | N $\$ 1.00$ |
| Average Winter (AW) | 180 Gallons | N $\$ 1.85$ |


| Severe Winter (SW) | 230 Gallons | N\$2.00 |
| :--- | :---: | :---: |
| Prolonged Winter (PW) | 250 Gallons | N\$3.00 |

Formulate a game model and employ the Minimax criterion technique to determine the gallons of oil Eufenia should stockpile at the current price of $N \$ 1$ per gallon to avoid wasted unused oil and to maximise his saving.
(17 Marks)

## QUESTION 3 [25 MARKS]

(a) A licensed mining company in possession of a natural resource field has 0.25 chance for diamond discovery. However, the company has the options to either mine the resource or to sell it the field to another mining company wishing to buy the land for $\$ 90,000,000$. The cost of mining by the licensee is $\$ 100,000,000$ with a revenue yield of $\$ 800,000,000$ if diamond is found. Formulate a game of strategy model to perform a mathematical decision analysis of the problem and determine the decision of the licensee, stating appropriate assumptions for your method.
(8 Marks)
(b) State the Maximum Likelihood Criterion and confirm the above decision obtained in (a) with the criterion.
(c) Assuming the company feels that the true chances of discovering diamond are likely to be between $15 \%$ and $35 \%$, define sensitivity analysis and the decision crossover point, and hence show that the decision is sensitive to these prior probabilities, providing an appropriate sketch to substantiate your decision analysis and conclusion.
(13 Marks)
QUESTION 4 [66 MARKS]
(a) Using the linear programming problem (LPP) approach in obtaining the solution of the game with the following payoff matrix:

$$
\left[\begin{array}{cccc}
0 & 1 & -1 & 2 \\
-1 & -3 & 0 & 0 \\
0 & 0 & -2 & 1 \\
0 & -2 & -3 & -1
\end{array}\right]
$$

Obtain the optimal mixed strategies for the two players and the value of the game, discussing your solutions.
(50 Marks)
(b) Consider the following Queueing System Data:

END OF EXAMINATION
TOTAL MARKS:161 (CONVERT TO 100\%)

