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|--|---|
| QUALIFICATIONS : <b>BACHELOR of SCIENCE HONOURS IN APPLIED MATHEMATICS</b><br><b>BACHELOR of SCIENCE HONOURS IN APPLIED STATISTICS</b> |   |
| QUALIFICATION CODES: <b>08BSMH, 08BSSH</b>   | LEVEL: <b>8</b>                             |
| COURSE: <b>APPLIED OPERATIONS RESEARCH</b>   | COURSE CODE: <b>AOR802S</b>                 |
| DATE: <b>JANUARY 2025</b>  | SESSION: <b>1</b>                           |
| DURATION: <b>3 HOURS</b>   | MARKS: <b>155</b> (To be converted to 100%) |

**SECOND OPPORTUNITY/SUPPLEMENTARY: EXAMINATION QUESTION PAPER**

**EXAMINER:** *Prof Sunday A. Reju*

**MODERATOR:** *Prof Oluwole D. Makinde*

**INSTRUCTIONS**

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.
6. Use of COMMA is NOT ALLOWED for a DECIMAL POINT.

**PERMISSIBLE MATERIALS**

1. Non-Programmable Calculator

**ATTACHMENTS**

NONE

**This paper consists of 4 pages including this front page.**

### QUESTION 1 [26 MARKS]

A construction company is bidding for the building of a new College Hostel or its Classroom Block or a combination of both. The construction company must submit a bid proposal, which costs money to prepare, and there are no guarantees that it will be awarded the contract. If the company bids on the Hostel, it has a 35% chance of getting the contract, and it expects to make \$162,000 net profit. However, if the company does not get the contract, it loses \$11,500. If the company bids on the Classroom Block, there is a 25% chance of getting the contract, and it would net \$140,000 in profit. However, if the company does not get the contract, it will lose \$5,750.

- (a) What should the construction company do? (10 Marks)
- (b) How sensitive to the estimate of the probability of the award of a contract is the decision (i):
- in either to build the Hostel or the Classroom Block? (6.5 Marks)
  - to the net profit for each case, if awarded the contract? (9.5 Marks)

### QUESTION 2 [63 MARKS]

(a) Using the linear programming problem (LPP) approach in obtaining the solution of the game with the following payoff matrix:

$$\begin{bmatrix} 0 & 1 & -1 & 2 \\ -1 & -3 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & -2 & -3 & -1 \end{bmatrix}$$

Obtain the optimal mixed strategies for the two players and the value of the game, discussing your solutions. (47 Marks)

(b) Consider the following Queueing System Data:

#### Queueing System for 2 Servers with Balking and Reneging

| Start Time                                 | Close Time  | Balk if queue length is or exceeds | Reneg if waiting time exceeds |
|--|-------------|------------------------------------|-------------------------------|
| 09:00                                      | 09:20       | 1                                  | 2 minutes                     |
| Interarrival Time Probability Distribution |             |                                    |                               |
| Probability                                | Lower Bound | Upper Bound                        | Interarrival Time (min)       |
| 0.45                                       | 0           | 0.45                               | 1                             |
| 0.25                                       | 0.45        | 0.7                                | 2                             |
| 0.1  | 0.7         | 0.8                                | 2                             |
| 0.2  | 0.8         | 1                                  | 1                             |
| Service Time Probability Distribution      |             |                                    |                               |
| Probability                                | Lower Bound | Upper Bound                        | Service Time (min)            |
| 0.3  | 0           | 0.3                                | 2                             |
| 0.35                                       | 0.3         | 0.65                               | 4                             |
| 0.35                                       | 0.65        | 1                                  | 6                             |

Obtain a Simulation Table for 5 customers using the following header: (16 Marks)



|       |              |          |            |       |          |          |
|-------|--------------|----------|------------|-------|----------|----------|
|       |              |          | Queue      |       |          |          |
| Cust  | Interarrival | Arrival  | Length     |       |          | Reneged  |
| #     | Time         | Time     | at Arrival | Balk? | Reneged? | Depart   |
|       | (min)        | (hr:min) | (# cust.)  |       |          | (hr:min) |
| start |              |          |            |       |          |          |
| 1     |              |          |            |       |          |          |
| ...   |              |          |            |       |          |          |

|       |         |           |          |           |          |           |          |          |
|-------|---------|-----------|----------|-----------|----------|-----------|----------|----------|
|       |         |           |          |           |          |           |          |          |
| Cust  | Service | Server #1 |          | Server #2 |          | Reneged   | Wait     | Total    |
| #     | Time    | Start     | End      | Start     | End      | Wait Time | Time     | Time     |
|       | (min)   | (hr:min)  | (hr:min) | (hr:min)  | (hr:min) | (hr:min)  | (hr:min) | (hr:min) |
| start |         |           |          |           |          |           |          |          |
| 1     |         |           |          |           |          |           |          |          |
| ...   |         |           |          |           |          |           |          |          |

### QUESTION 3 [34 MARKS]

(a) Provide a comprehensive definition of a Decision tree and hence diagrammatically show its basic characteristic components. (12 Marks)

(b)

(i) What is the Kendall's classification of Queuing Systems? (3 Marks)

Discuss specifically the M/M/1 queuing system and the process  $N(t)$  describing its state at time  $t$  as a birth-death process. Provide its state independent parameter equations and define its Traffic Intensity. (4 Marks)

(ii) Consider a drive-in banking service modelled as an M/M/1 queuing system with customer arrival rate of 2 per minute. It is desired to have fewer than 5 customers line up 99% of the time. How fast should the service rate be? (6 Marks)

(c) Trucks arrive at garage for a stop-over service according to a Poisson process at a rate of one per every 13 minutes, and the garage service time is an exponential rate variable with mean 9 minutes.

(i) Find the average number  $L$  of trucks, the average time  $W$  a truck spends in the garage, and the average time  $W_q$  a truck spends in waiting for service. (5 Marks)

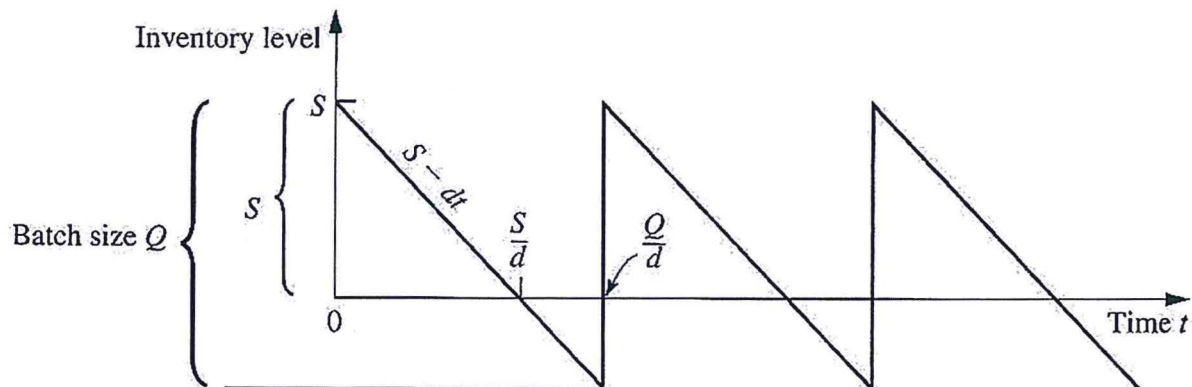
(ii) Due to increased traffic, suppose that the arrival rate of the trucks increases by 5%. Find the corresponding changes in  $L$ ,  $W$ , and  $W_q$ . (4 Marks)

(iii) Discuss your observations. (1 Mark)

#### QUESTION 4 [32 MARKS]

(a) Consider the following diagram for inventory with shortage:

(22 Marks)



Given  $p$  as the shortage cost per unit short per unit of time short and  $S$  as the inventory level just after a batch of  $Q$  units is added to inventory, obtain the expressions for:

- (i) Shortage per cycle (ii) Total cost per cycle (iii)  $Q^*$  (iv)  $S^*$
- (v)  $t^*$  (vi)  $(Q^* - S^*)$  (vii) The fraction of time that no shortage exists

(b)

A computer firm produces its own power units which are used in the production of the desktop computers. The computer machines are assembled on a continuous production line at a rate of 8,000 monthly, with one power unit per desktop. The power units are produced in batches and therefore placed into inventory until they are needed for assembly into desktop machines on the production line. The firm is interested in determining when to produce a batch of units and how many of them to produce in each batch, if 24,000 power units are produced in each production run with the following costs: (10 Marks)

- (i) Holding cost = \$ 0.30.
- (ii) Unit cost of a single power unit (excluding the setup cost), independent of the batch size produced = \$ 10.
- (iii) The cost of each unit that is not available = \$ 1.10, per month.

From your expressions in (a), obtain the optimum number of units per order ( $Q^*$ ), optimum inventory level just after a batch of  $Q$  units is added to inventory ( $S^*$ ), optimum cycle length ( $t^*$ ) and the maximum shortage ( $Q^* - S^*$ ).

END OF EXAMINATION

TOTAL MARKS = 155