



**NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY**

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QUALIFICATION: BACHELOR OF SCIENCE IN APPLIED MATHEMATICS AND STATISTICS	
QUALIFICATION CODE: 07BAMS	LEVEL: 7
COURSE : DESIGN AND ANALYSIS OF EXPERIMENT	COURSE CODE: DAE702S
DATE : NOVEMBER 2023	SESSION: 1
DURATION : 3 HOURS	MARKS: 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER

EXAMINER	Dr. Jacob Ong'ala
MODERATOR	Dr Petrus Iiyambo

INSTRUCTION

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS

1. Non-Programmable Calculator

ATTACHEMENTS

1. F distribution table
2. OC Curves for the Fixed model Analysis of Variance
3. t distribution table

THIS QUESTION PAPER CONSISTS OF 4 PAGES
(including the front page)

QUESTION ONE - 26 MARKS

- (a) Discuss the basic principles of experimental design [6 mks]
- (b) Rats were given one of four different diets at random, and the response measure was liver weight as a percentage of body weight. The responses were:

Treatment			
1	2	3	4
3.52	3.47	3.54	3.74
3.36	3.73	3.52	3.83
3.57	3.38	3.61	3.87
4.19	3.87	3.76	4.08
3.88	3.69	3.65	4.31
3.76	3.51	3.51	3.98
3.94	3.35		3.86
	3.64		3.71

- (i) Formulate the hypothesis when fixed effect model is assumed [2 mks]
- (ii) Compute the Analysis of Variance table for these data. What would you conclude about the four diets? [13 mks]
- (iii) Construct a 99 percent confidence interval estimate of the mean response for treatment 3. [5 mks]

QUESTION TWO - 28 MARKS

- (a) In many integrated circuit manufacturing steps, wafers are completely coated with a layer of material such as silicon dioxide or a metal. The unwanted material is then selectively removed by etching through a mask, thereby creating circuit patterns, electrical interconnects, and areas in which diffusions or metal depositions are to be made. Energy is supplied by a radio frequency (RF) generator causing plasma to be generated in the gap between the electrodes. The chemical species in the plasma are determined by the particular gases used. An engineer is interested in investigating the relationship between the RF power setting and the etch rate for this tool. The objective of an experiment is to model the relationship between etch rate and RF power, and to specify the power setting that will give a desired target etch rate. She wants to test four levels of RF power: 160W, 180W, 200W, and 220W. Past experience indicate that mean etch rate for each RF power are $\mu_1 = 573$, $\mu_2 = 593$, $\mu_3 = 629$ and $\mu_4 = 681$. Suppose that the experimenter is interested in rejecting the null hypothesis with a probability of at least 0.90 and she feels that the standard deviation of etch rate at any particular level of power will be no larger than $\sigma = 30$ A/min. and she plans to use $\alpha = 0.01$.

(Note: Use the Operating Characteristic Curves for the Fixed Effects Model Analysis of Variance)

- (i) Compute the treatment effect for each RF power using the above information [2 mks]
- (ii) Starting with an initial sample 3, determine the recommended minimum sample size (n^*) for each treatments that is required to obtain power of 0.90 [11 mks]

- (b) Suppose you want to determine whether the brand of laundry detergent used and the temperature affects the amount of dirt removed from your laundry. To this end, you buy two different brand of detergent (“Super” and “Best”) and choose three different temperature levels (“cold”, “warm”, and “hot”). Then you divide your laundry randomly into $6 \times r$ piles of equal size and assign each $r=4$ piles into the combination of (“Super” and “Best”) and (“cold”, “warm”, and “hot”). The amounts Y_{ijk} of dirt removed when washing sub pile $k(k = 1, 2, 3, 4)$ with detergent $i(i = 1, 2)$ at temperature $j(j = 1, 2, 3)$ are recorded in the table below.

	Cold	Warm	Hot
Super	4,5,6,5	7,9,8,12	10,12,11,9
Best	6,6,4,4	13,15,12,12	12,13,10,13

- (i) Formulate the hypotheses for this experiment [2 mks]
(ii) Test the hypothesis above using Analysis of Variance [10 mks]
(iii) What do you conclude from the ANOVA results above [3 mks]

QUESTION THREE - 29 MARKS

- (a) The ANOVA from a randomized complete block experiment output is shown below.

Source	DF	SS	MS	F	P
Treatment	4	1010.56	?	29.84	?
Block	?	?	64.765	?	?
Error	20	169.33	?		
Total	29	1503.71			

- (i) Fill in the blanks [4 mks]
(ii) How many blocks were used in this experiment? [1 mks]
(iii) What conclusions can you draw? [2 mks]
- (b) Three different washing solutions are being compared to study their effectiveness in retarding bacteria growth in 5-gallon milk containers. The analysis is done in a laboratory, and only three trials can be run on any day. Because days could represent a potential source of variability, the experimenter decides to use a randomized block design. Observations are taken for four days, and the data are shown below

Solution	Days			
	1	2	3	4
1	13	22	18	39
2	16	24	17	44
3	5	4	1	22

- (i) Analyze the data from this experiment. (use $\alpha = 0.05$) [13 mks]
(ii) Use the Fisher LSD method to make comparisons among the three Solutions at $\alpha = 0.05$ [9 mks]

QUESTION FOUR - 17 MARKS

An industrial engineer is investigating the effect of four assembly methods (A, B, C, D) on the assembly time for a color television component. Four operators are selected for the study. Furthermore, the engineer knows that each assembly method produces such fatigue that the time required for the last assembly may be greater than the time required for the first, regardless of the method. That is, a trend develops in the required assembly time. To account for this source of variability, the engineer uses the Latin square design shown below. Analyze the data from this experiment ($\alpha = 0.05$) and draw appropriate conclusions. (*In this question, use coded data*)

Order of Assembly	Operator			
	1	2	3	4
1	C = 10	D = 14	A = 7	B = 8
2	B = 7	C = 18	D = 11	A = 8
3	A = 5	B = 10	C = 11	D = 9
4	D = 10	A = 10	B = 12	C = 14

- END OF QUESTIONS -

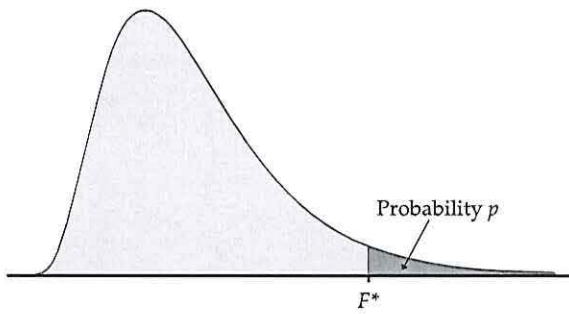


Table entry for p is the critical value F^* with probability p lying to its right.

TABLE E
F critical values

		Degrees of freedom in the numerator									
p		1	2	3	4	5	6	7	8	9	
Degrees of freedom in the denominator	1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
		.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
		.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
		.010	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
	2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
		.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
		.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
		.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
		.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
		.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
		.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
	4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
		.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
		.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
		.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
		.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
		.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
		.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	
	.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	
	.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	
	.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	
7	.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	
	.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	
	.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	
	.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	
	.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33	

Table entry for p is the critical value F^* with probability p lying to its right.

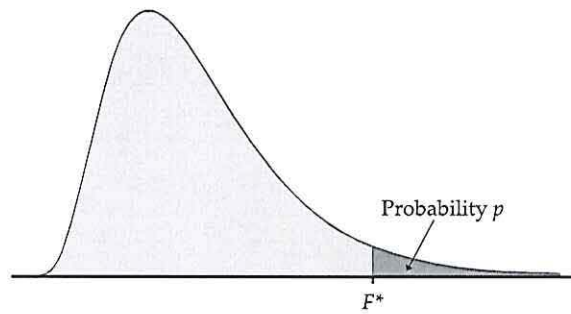


TABLE E										
F critical values (continued)										
Degrees of freedom in the numerator										
10	12	15	20	25	30	40	50	60	120	1000
60.19	60.71	61.22	61.74	62.05	62.26	62.53	62.69	62.79	63.06	63.30
241.88	243.91	245.95	248.01	249.26	250.10	251.14	251.77	252.20	253.25	254.19
968.63	976.71	984.87	993.10	998.08	1001.4	1005.6	1008.1	1009.8	1014.0	1017.7
6055.8	6106.3	6157.3	6208.7	6239.8	6260.6	6286.8	6302.5	6313.0	6339.4	6362.7
605621	610668	615764	620908	624017	626099	628712	630285	631337	633972	636301
9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.49
19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.48	19.49	19.49
39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.48	39.49	39.50
99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.48	99.49	99.50
999.40	999.42	999.43	999.45	999.46	999.47	999.47	999.48	999.48	999.49	999.50
5.23	5.22	5.20	5.18	5.17	5.17	5.16	5.15	5.15	5.14	5.13
8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.58	8.57	8.55	8.53
14.42	14.34	14.25	14.17	14.12	14.08	14.04	14.01	13.99	13.95	13.91
27.23	27.05	26.87	26.69	26.58	26.50	26.41	26.35	26.32	26.22	26.14
129.25	128.32	127.37	126.42	125.84	125.45	124.96	124.66	124.47	123.97	123.53
3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.80	3.79	3.78	3.76
5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.63
8.84	8.75	8.66	8.56	8.50	8.46	8.41	8.38	8.36	8.31	8.26
14.55	14.37	14.20	14.02	13.91	13.84	13.75	13.69	13.65	13.56	13.47
48.05	47.41	46.76	46.10	45.70	45.43	45.09	44.88	44.75	44.40	44.09
3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.12	3.11
4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.44	4.43	4.40	4.37
6.62	6.52	6.43	6.33	6.27	6.23	6.18	6.14	6.12	6.07	6.02
10.05	9.89	9.72	9.55	9.45	9.38	9.29	9.24	9.20	9.11	9.03
26.92	26.42	25.91	25.39	25.08	24.87	24.60	24.44	24.33	24.06	23.82
2.94	2.90	2.87	2.84	2.81	2.80	2.78	2.77	2.76	2.74	2.72
4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.75	3.74	3.70	3.67
5.46	5.37	5.27	5.17	5.11	5.07	5.01	4.98	4.96	4.90	4.86
7.87	7.72	7.56	7.40	7.30	7.23	7.14	7.09	7.06	6.97	6.89
18.41	17.99	17.56	17.12	16.85	16.67	16.44	16.31	16.21	15.98	15.77
2.70	2.67	2.63	2.59	2.57	2.56	2.54	2.52	2.51	2.49	2.47
3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.32	3.30	3.27	3.23
4.76	4.67	4.57	4.47	4.40	4.36	4.31	4.28	4.25	4.20	4.15
6.62	6.47	6.31	6.16	6.06	5.99	5.91	5.86	5.82	5.74	5.66
14.08	13.71	13.32	12.93	12.69	12.53	12.33	12.20	12.12	11.91	11.72

(Continued)

TABLE E
F critical values (continued)

		Degrees of freedom in the numerator										
<i>p</i>		1	2	3	4	5	6	7	8	9		
Degrees of freedom in the denominator	8	.100	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	
		.050	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	
		.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	
		.010	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	
		.001	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77	
		9	.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
		.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	
		.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	
		.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	
		.001	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11	
		10	.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
		.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	
		.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	
		.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	
		.001	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96	
		11	.100	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
		.050	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	
	.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59		
	.010	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63		
	.001	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12		
	12	.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	
	.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80		
	.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44		
	.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39		
	.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48		
	13	.100	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	
	.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71		
	.025	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31		
	.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19		
	.001	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98		
	14	.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	
	.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65		
	.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21		
	.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03		
	.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58		
	15	.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	
	.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59		
	.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12		
	.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89		
	.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26		
	16	.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	
	.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54		
	.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05		
	.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78		
	.001	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98		
	17	.100	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	
	.050	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49		
	.025	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98		
	.010	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68		
	.001	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75		

TABLE E

F critical values (continued)

Degrees of freedom in the numerator										
10	12	15	20	25	30	40	50	60	120	1000
2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.30
3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.02	3.01	2.97	2.93
4.30	4.20	4.10	4.00	3.94	3.89	3.84	3.81	3.78	3.73	3.68
5.81	5.67	5.52	5.36	5.26	5.20	5.12	5.07	5.03	4.95	4.87
11.54	11.19	10.84	10.48	10.26	10.11	9.92	9.80	9.73	9.53	9.36
2.42	2.38	2.34	2.30	2.27	2.25	2.23	2.22	2.21	2.18	2.16
3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.80	2.79	2.75	2.71
3.96	3.87	3.77	3.67	3.60	3.56	3.51	3.47	3.45	3.39	3.34
5.26	5.11	4.96	4.81	4.71	4.65	4.57	4.52	4.48	4.40	4.32
9.89	9.57	9.24	8.90	8.69	8.55	8.37	8.26	8.19	8.00	7.84
2.32	2.28	2.24	2.20	2.17	2.16	2.13	2.12	2.11	2.08	2.06
2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.64	2.62	2.58	2.54
3.72	3.62	3.52	3.42	3.35	3.31	3.26	3.22	3.20	3.14	3.09
4.85	4.71	4.56	4.41	4.31	4.25	4.17	4.12	4.08	4.00	3.92
8.75	8.45	8.13	7.80	7.60	7.47	7.30	7.19	7.12	6.94	6.78
2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	1.98
2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.51	2.49	2.45	2.41
3.53	3.43	3.33	3.23	3.16	3.12	3.06	3.03	3.00	2.94	2.89
4.54	4.40	4.25	4.10	4.01	3.94	3.86	3.81	3.78	3.69	3.61
7.92	7.63	7.32	7.01	6.81	6.68	6.52	6.42	6.35	6.18	6.02
2.19	2.15	2.10	2.06	2.03	2.01	1.99	1.97	1.96	1.93	1.91
2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.40	2.38	2.34	2.30
3.37	3.28	3.18	3.07	3.01	2.96	2.91	2.87	2.85	2.79	2.73
4.30	4.16	4.01	3.86	3.76	3.70	3.62	3.57	3.54	3.45	3.37
7.29	7.00	6.71	6.40	6.22	6.09	5.93	5.83	5.76	5.59	5.44
2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.85
2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.31	2.30	2.25	2.21
3.25	3.15	3.05	2.95	2.88	2.84	2.78	2.74	2.72	2.66	2.60
4.10	3.96	3.82	3.66	3.57	3.51	3.43	3.38	3.34	3.25	3.18
6.80	6.52	6.23	5.93	5.75	5.63	5.47	5.37	5.30	5.14	4.99
2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.87	1.86	1.83	1.80
2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.24	2.22	2.18	2.14
3.15	3.05	2.95	2.84	2.78	2.73	2.67	2.64	2.61	2.55	2.50
3.94	3.80	3.66	3.51	3.41	3.35	3.27	3.22	3.18	3.09	3.02
6.40	6.13	5.85	5.56	5.38	5.25	5.10	5.00	4.94	4.77	4.62
2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.83	1.82	1.79	1.76
2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.18	2.16	2.11	2.07
3.06	2.96	2.86	2.76	2.69	2.64	2.59	2.55	2.52	2.46	2.40
3.80	3.67	3.52	3.37	3.28	3.21	3.13	3.08	3.05	2.96	2.88
6.08	5.81	5.54	5.25	5.07	4.95	4.80	4.70	4.64	4.47	4.33
2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.79	1.78	1.75	1.72
2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.12	2.11	2.06	2.02
2.99	2.89	2.79	2.68	2.61	2.57	2.51	2.47	2.45	2.38	2.32
3.69	3.55	3.41	3.26	3.16	3.10	3.02	2.97	2.93	2.84	2.76
5.81	5.55	5.27	4.99	4.82	4.70	4.54	4.45	4.39	4.23	4.08
2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.76	1.75	1.72	1.69
2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.08	2.06	2.01	1.97
2.92	2.82	2.72	2.62	2.55	2.50	2.44	2.41	2.38	2.32	2.26
3.59	3.46	3.31	3.16	3.07	3.00	2.92	2.87	2.83	2.75	2.66
5.58	5.32	5.05	4.78	4.60	4.48	4.33	4.24	4.18	4.02	3.87

(Continued)

TABLE E
F critical values (continued)

		Degrees of freedom in the numerator										
<i>p</i>		1	2	3	4	5	6	7	8	9		
Degrees of freedom in the denominator	18	.100	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	
		.050	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	
		.025	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	
		.010	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	
		.001	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56	
		19	.100	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98
			.050	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
			.025	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88
			.010	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
			.001	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39
		20	.100	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
			.050	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
			.025	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
			.010	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
			.001	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24
		21	.100	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95
			.050	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
			.025	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80
			.010	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
			.001	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11
	22	.100	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	
		.050	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	
		.025	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	
		.010	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	
		.001	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99	
	23	.100	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	
		.050	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	
		.025	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	
		.010	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	
		.001	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89	
	24	.100	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	
		.050	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	
		.025	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	
		.010	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	
		.001	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80	
	25	.100	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	
		.050	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	
		.025	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	
		.010	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	
		.001	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71	
	26	.100	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	
		.050	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	
		.025	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	
		.010	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	
		.001	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64	
	27	.100	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	
		.050	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	
		.025	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	
		.010	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	
		.001	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57	

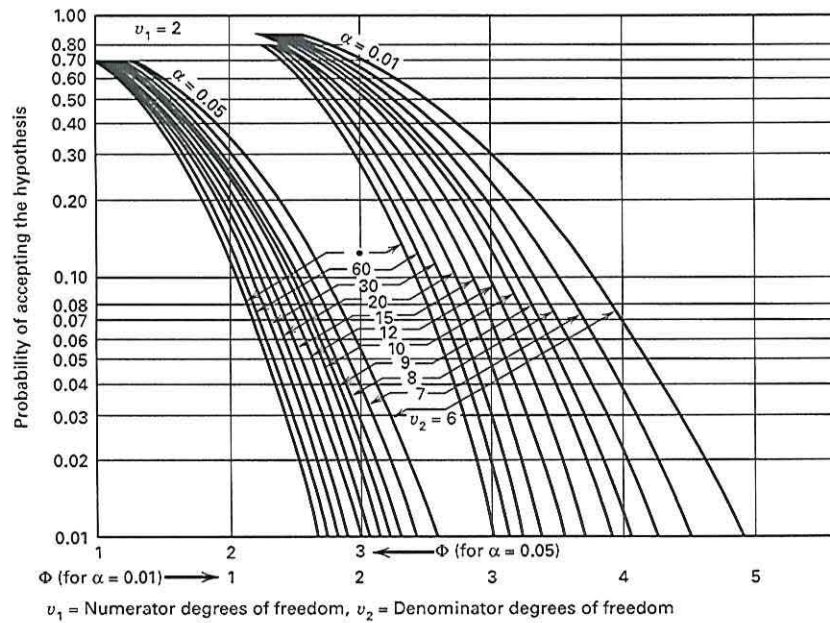
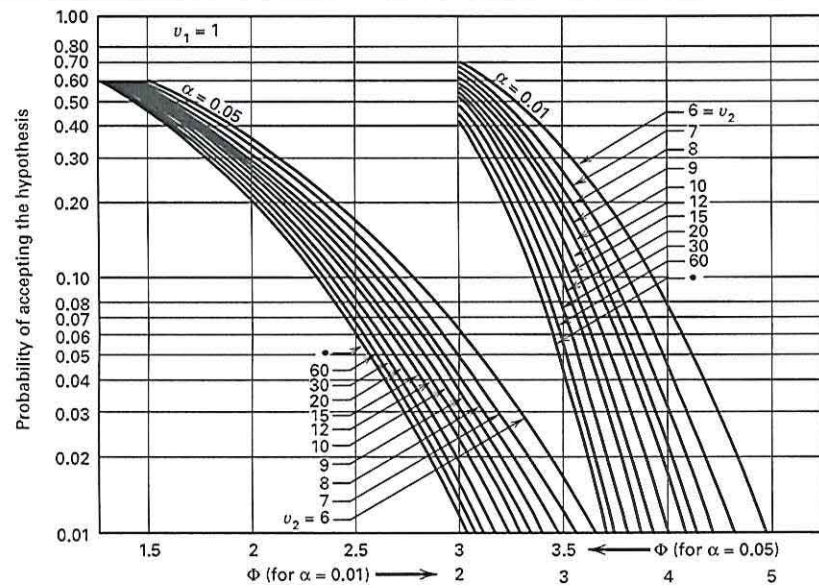
TABLE E

F critical values (continued)

Degrees of freedom in the numerator										
10	12	15	20	25	30	40	50	60	120	1000
1.98	1.93	1.89	1.84	1.80	1.78	1.75	1.74	1.72	1.69	1.66
2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.04	2.02	1.97	1.92
2.87	2.77	2.67	2.56	2.49	2.44	2.38	2.35	2.32	2.26	2.20
3.51	3.37	3.23	3.08	2.98	2.92	2.84	2.78	2.75	2.66	2.58
5.39	5.13	4.87	4.59	4.42	4.30	4.15	4.06	4.00	3.84	3.69
1.96	1.91	1.86	1.81	1.78	1.76	1.73	1.71	1.70	1.67	1.64
2.38	2.31	2.23	2.16	2.11	2.07	2.03	2.00	1.98	1.93	1.88
2.82	2.72	2.62	2.51	2.44	2.39	2.33	2.30	2.27	2.20	2.14
3.43	3.30	3.15	3.00	2.91	2.84	2.76	2.71	2.67	2.58	2.50
5.22	4.97	4.70	4.43	4.26	4.14	3.99	3.90	3.84	3.68	3.53
1.94	1.89	1.84	1.79	1.76	1.74	1.71	1.69	1.68	1.64	1.61
2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.97	1.95	1.90	1.85
2.77	2.68	2.57	2.46	2.40	2.35	2.29	2.25	2.22	2.16	2.09
3.37	3.23	3.09	2.94	2.84	2.78	2.69	2.64	2.61	2.52	2.43
5.08	4.82	4.56	4.29	4.12	4.00	3.86	3.77	3.70	3.54	3.40
1.92	1.87	1.83	1.78	1.74	1.72	1.69	1.67	1.66	1.62	1.59
2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.94	1.92	1.87	1.82
2.73	2.64	2.53	2.42	2.36	2.31	2.25	2.21	2.18	2.11	2.05
3.31	3.17	3.03	2.88	2.79	2.72	2.64	2.58	2.55	2.46	2.37
4.95	4.70	4.44	4.17	4.00	3.88	3.74	3.64	3.58	3.42	3.28
1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.60	1.57
2.30	2.23	2.15	2.07	2.02	1.98	1.94	1.91	1.89	1.84	1.79
2.70	2.60	2.50	2.39	2.32	2.27	2.21	2.17	2.14	2.08	2.01
3.26	3.12	2.98	2.83	2.73	2.67	2.58	2.53	2.50	2.40	2.32
4.83	4.58	4.33	4.06	3.89	3.78	3.63	3.54	3.48	3.32	3.17
1.89	1.84	1.80	1.74	1.71	1.69	1.66	1.64	1.62	1.59	1.55
2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.88	1.86	1.81	1.76
2.67	2.57	2.47	2.36	2.29	2.24	2.18	2.14	2.11	2.04	1.98
3.21	3.07	2.93	2.78	2.69	2.62	2.54	2.48	2.45	2.35	2.27
4.73	4.48	4.23	3.96	3.79	3.68	3.53	3.44	3.38	3.22	3.08
1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.57	1.54
2.25	2.18	2.11	2.03	1.97	1.94	1.89	1.86	1.84	1.79	1.74
2.64	2.54	2.44	2.33	2.26	2.21	2.15	2.11	2.08	2.01	1.94
3.17	3.03	2.89	2.74	2.64	2.58	2.49	2.44	2.40	2.31	2.22
4.64	4.39	4.14	3.87	3.71	3.59	3.45	3.36	3.29	3.14	2.99
1.87	1.82	1.77	1.72	1.68	1.66	1.63	1.61	1.59	1.56	1.52
2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.84	1.82	1.77	1.72
2.61	2.51	2.41	2.30	2.23	2.18	2.12	2.08	2.05	1.98	1.91
3.13	2.99	2.85	2.70	2.60	2.54	2.45	2.40	2.36	2.27	2.18
4.56	4.31	4.06	3.79	3.63	3.52	3.37	3.28	3.22	3.06	2.91
1.86	1.81	1.76	1.71	1.67	1.65	1.61	1.59	1.58	1.54	1.51
2.22	2.15	2.07	1.99	1.94	1.90	1.85	1.82	1.80	1.75	1.70
2.59	2.49	2.39	2.28	2.21	2.16	2.09	2.05	2.03	1.95	1.89
3.09	2.96	2.81	2.66	2.57	2.50	2.42	2.36	2.33	2.23	2.14
4.48	4.24	3.99	3.72	3.56	3.44	3.30	3.21	3.15	2.99	2.84
1.85	1.80	1.75	1.70	1.66	1.64	1.60	1.58	1.57	1.53	1.50
2.20	2.13	2.06	1.97	1.92	1.88	1.84	1.81	1.79	1.73	1.68
2.57	2.47	2.36	2.25	2.18	2.13	2.07	2.03	2.00	1.93	1.86
3.06	2.93	2.78	2.63	2.54	2.47	2.38	2.33	2.29	2.20	2.11
4.41	4.17	3.92	3.66	3.49	3.38	3.23	3.14	3.08	2.92	2.78

(Continued)

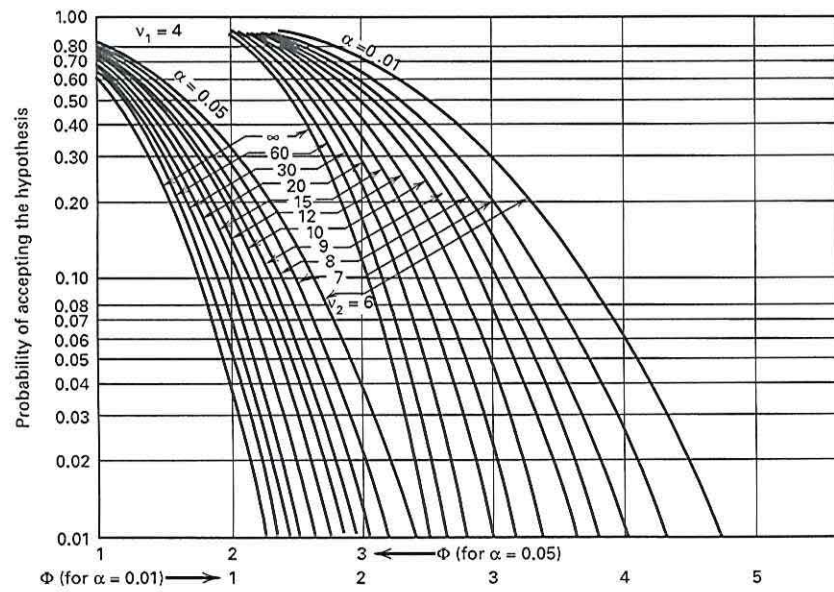
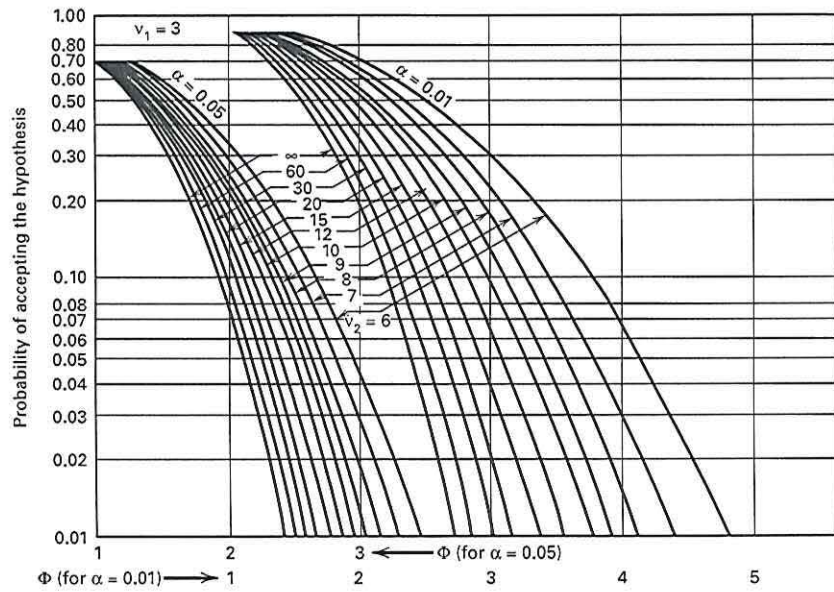
V Operating Characteristic Curves for the Fixed Effects Model Analysis of Variance^a



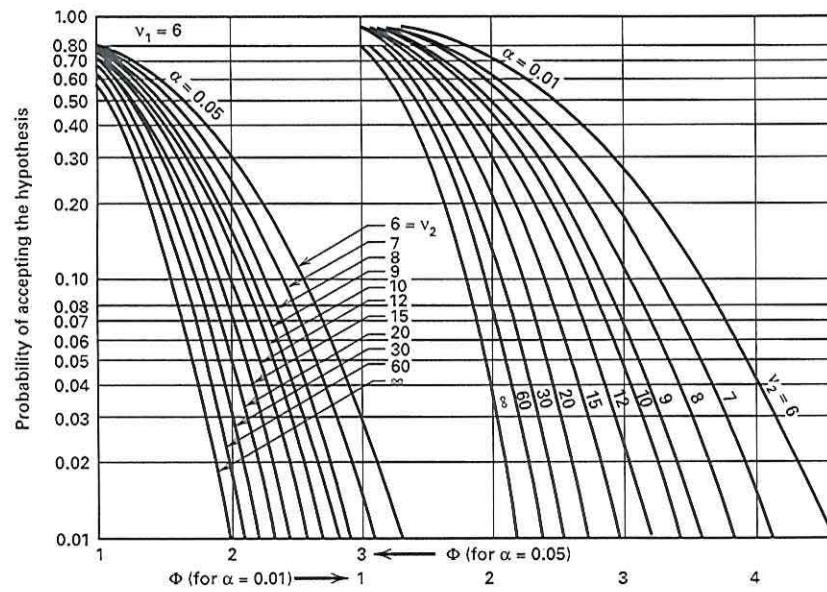
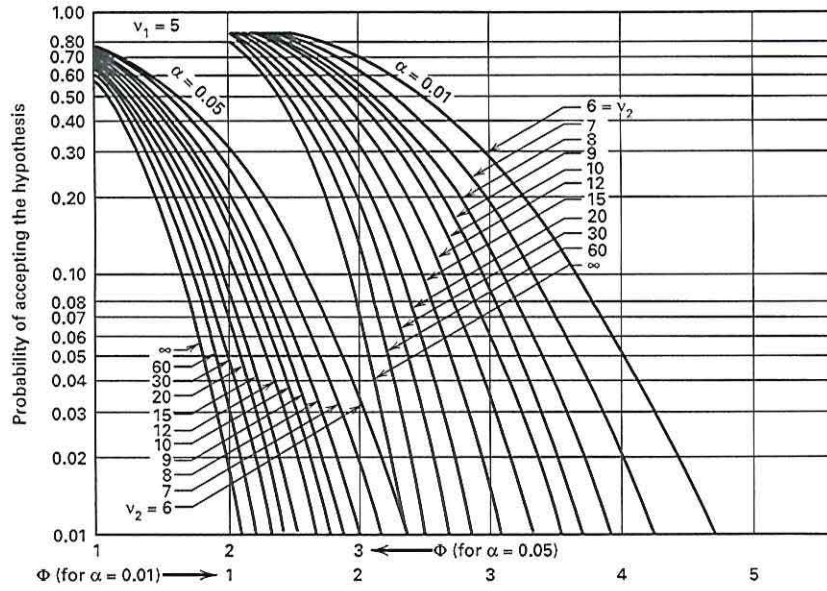
v_1 = Numerator degrees of freedom, v_2 = Denominator degrees of freedom

^aAdapted with permission from *Biometrika Tables for Statisticians*, Vol. 2, by E. S. Pearson and H. O. Hartley, Cambridge University Press, Cambridge, 1972.

V Operating Characteristic Curves for the Fixed Effects Model Analysis of Variance (Continued)



V Operating Characteristic Curves for the Fixed Effects Model Analysis of Variance (Continued)



V Operating Characteristic Curves for the Fixed Effects Model Analysis of Variance (Continued)

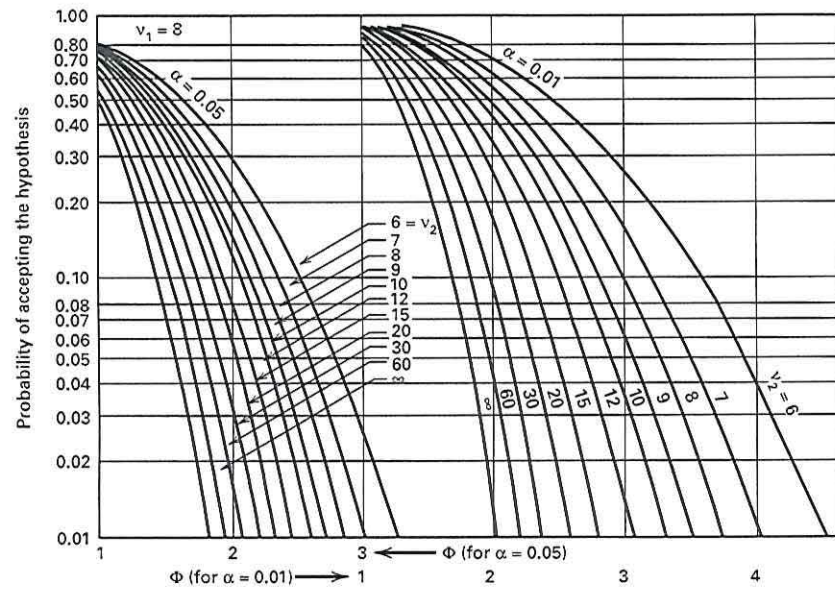
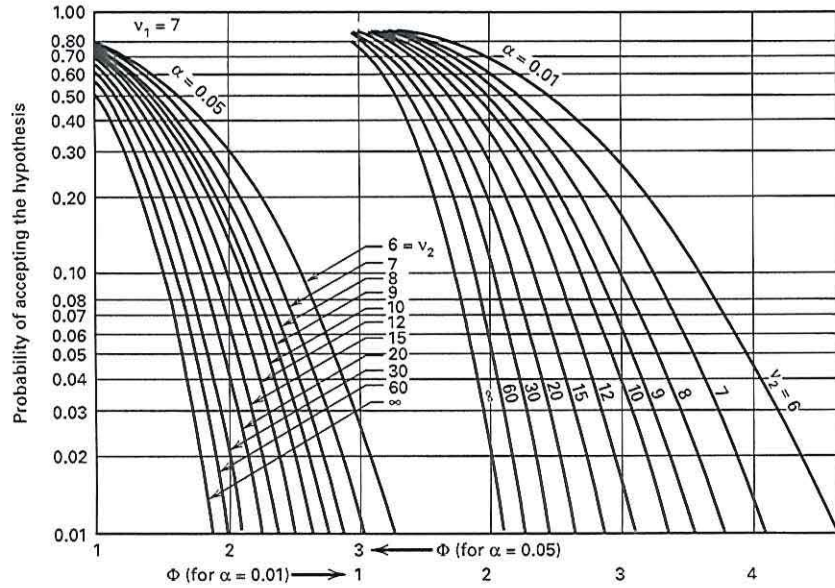


Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^* .

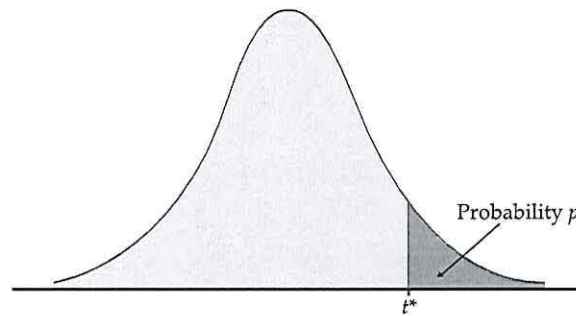


TABLE D

t distribution critical values

df	Upper-tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											