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QUALIFICATION : **BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS**	
QUALIFICATION CODE: **07BSAM**	LEVEL: **7**
COURSE: **NUMERICAL METHODS 2**	COURSE CODE: **NUM702S**
DATE: **NOVEMBER 2023**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

FIRST OPPORTUNITY: EXAMINATION QUESTION PAPER

EXAMINER: *Dr S.N. NEOSSI-NGUETCHUE*

MODERATOR: *Prof S.S. MOTSA*

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Show clearly all the steps used in the calculations. All numerical results must be given using 5 decimals where necessary unless mentioned otherwise.
6. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

ATTACHEMENTS

None

This paper consists of 3 pages including this front page

Problem 1 [19 Marks]

1-1. Find the Padé approximation $R_{2,2}(x)$ for $f(x) = \ln(1+x)/x$ starting with the MacLaurin expansion

$$f(x) = 1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \frac{x^4}{5} - \cdots . \qquad [12]$$

1-2. Use the result in **1-1.** to establish $\ln(1+x) \approx R_{3,2} = \dfrac{30x + 21x^2 + x^3}{30 + 36x + 9x^2}$ and express $R_{3,2}$ in continued fraction form. [7]

Problem 2 [30 Marks]

For any non negative interger n we define Chebyshev polynomial of the first kind as

$$T_n(x) = \cos(n\theta), \quad \text{where } \theta = \arccos(x), \text{ for } x \in [-1, 1].$$

2-1. Show the following property: [5]

$$T_n \text{ has } n \text{ distinct zeros } x_k \in [-1, 1] : x_k = \cos\left(\frac{(2k+1)\pi}{2n}\right) \text{ for } 0 \le k \le n-1.$$

2-2. Compute the expressions of the first five Chebyshev polynomials of the first kind T_0, T_1, T_2, T_3 and T_4. [3]

2-3. Given the trucated power series $f(x) = 1 + 2x - x^3 + 3x^4$.
(i) Economise the power series $f(x)$. [3]
(ii) Find the Chebyshev series for $f(x)$. [5]

2-4. (i) Show that the following function f is even and use an appropriate result to find its Fourier series

$$f(x) = \begin{cases} \dfrac{\pi}{2} + x, & \text{for } -\pi \le x < 0, \\ \dfrac{\pi}{2} - x, & \text{for } 0 \le x < \pi. \end{cases} \qquad [12]$$

(ii) Set $x = 0$ and conclude that $\dfrac{\pi^2}{8} = 1 + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \dfrac{1}{7^2} + \cdots$. [2]

Problem 3 [27 Marks]

3-1. Given the integral

$$\int_0^3 \frac{\sin(2x)}{1+x^5} dx = 0.6717578646 \cdots$$

3-1-1. Compute $T(J) = R(J, 0)$ for $J = 0, 1, 2, 3$ using the sequential trapezoidal rule. [10]

3-1-2. Use the results in **3-1-1.** and Romberg's rule to compute the values for the sequential Simpson rule $\{R(J, 1)\}$, sequential Boole rule $\{R(J, 2)\}$ and the third impprovement $\{R(J, 3)\}$. Display your results in a tabular form. [12]

3-2. State the three-point Gaussian Rule for a continuous function f on the interval $[-1, 1]$ and show that the rule is exact for $f(x) = 5x^4$. [5]

Problem 4 [24 Marks]

4-1. The matrix A and its inverse are A^{-1} are given below

$$A = \begin{bmatrix} 1/2 & -1 \\ -1 & 1 \end{bmatrix}, \qquad A^{-1} = \begin{bmatrix} -2 & -2 \\ -2 & -1 \end{bmatrix}.$$

• Use the power method to find the eigenvalue of the matrix A with the smallest absolute value. Start with the vector $\mathbf{x}^{(0)} = (1, 0)^T$ and perform two iterations. [6]

4-2. Use Jacobi's method to find the eigenpairs of the matrix

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

[18]

God bless you !!!