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QUALIFICATION : **BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS**	
QUALIFICATION CODE: **07BSAM**	LEVEL: **5**
COURSE: **LINEAR ALGEBRA 1**	COURSE CODE: **LIA502S**
DATE: **NOVEMBER 2023**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

FIRST OPPORTUNITY: QUESTION PAPER

EXAMINER:	MR GABRIEL S MBOKOMA, DR NEGA CHERE
MODERATOR:	DR DAVID IIYAMBO

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

Non-Programmable Calculator

This paper consists of 3 pages including this front page.

Question 1

1.1 Given that $\mathbf{u} = (6 - x, 4 - y)$ and $\mathbf{v} = (x - 4, y + 2)$ are vectors in $\mathbb{R}^2$, such that $\mathbf{u} = \mathbf{v}$, solve for x and y? [4]

1.2 Determine a unit vector perpendicular to both of the vectors $\mathbf{A} = \mathbf{c} + \mathbf{d}$ and $\mathbf{B} = \mathbf{c} - \mathbf{d}$, where $\mathbf{c} = 3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{d} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$. [7]

1.3 Consider the vectors $\mathbf{z} = (3 + 4i, 2 - i)$ and $\mathbf{w} = (1 + 3i, 1 - 2i)$ in $\mathbb{C}^2$. Determine whether $\mathbf{z}$ and $\mathbf{w}$ are orthogonal. [6]

1.4 Prove that if $\mathbf{x}$ and $\mathbf{y}$ are orthogonal vectors in $\mathbb{R}^n$, then show that

$$\|\mathbf{x} + \mathbf{y}\|^2 = \|\mathbf{x}\|^2 + \|\mathbf{y}\|^2.$$

[6]

Question 2

2.1 Write down a 4×4 matrix whose ij^{th} entry is given by $a_{ij} = \frac{1}{ij+1}$, and comment on your matrix. [6]

2.2 Let A be a square matrix. State what is meant by each of the following statements.

(a) A is symmetric [1]

(b) A is orthogonal [1]

(c) A is skew-symmetric [1]

2.3 Consider the matrix $A = \begin{pmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{pmatrix}$.

a) Use the *Cofactor expansion method along the second column* to evaluate the determinant of A. [7]

b) Is A invertible? If it is, Use the Gauss-Jordan Elimination method to find A^{-1}. [14]

c) Find $\det\left(3(2A)^{-1}\right)$. [6]

Question 3

Determine whether or not the vector (-1,1,5) is a linear combination of the vectors (1,2,3), (0,1,4) and (2,3,6). [15]

Question 4

a) Prove that a vector space cannot have more than one zero vector. [6]

b) Let M_{nn} be a vector space whose elements are all the $n \times n$ matrices, with the usual addition and scalar multiplication for matrices. Determine whether the following set is a subspace of M_{nn}.

$$S = \{A \in M_{nn} \mid tr(A) = 0\}$$

[11]

c) Prove or disprove that if U and W are subspaces of a vector space V, then $U \cap W$ is also a subspace of V. [9]
