NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

Faculty of Health, Natural Resources and Applied Sciences

School of Natural and Applied Sciences

Department of Mathematics, Statistics and Actuarial Science

13 Jackson Kaujeua Street
Private Bag 13388
Windhoek
NAMIBIA

T: +264 61 207 2913
E: msas@nust.na
W: www.nust.na

QUALIFICATION : **BACHELOR OF SCIENCE IN APPLIED MATHEMATICS AND STATISTICS**	
QUALIFICATION CODE: **07BSAM; 07BSOC**	LEVEL: **6**
COURSE: **LINEAR ALGEBRA 2**	COURSE CODE: **LIA601S**
DATE: **NOVEMBER 2023**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

FIRST OPPORTUNITY EXAMINATION: QUESTION PAPER

EXAMINER: DR. NEGA CHERE

MODERATOR: DR. DAVID IIYAMBO

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly with black or blue ink pen.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

ATTACHMENTS:

NONE

This paper consists of 3 pages including this front page.

Part I: True or false questions.

For each of the following questions, state whether it is true or false. Justify your answer.

1. If $T : P_3 \to P_3$ is a linear transformation, then T is an isomorphism. (3)
2. If the characterstic equation of a matrix A is given by $p(\lambda) = \lambda^2(\lambda - 1)(\lambda - 2)^3$, then the size of matrix A is 6×6. (2)
3. Let A be an n x n matrix. If A has fewer than n distinct eigenvalues then A is not diagonalizable. (3)
4. If q is a quadratic form on a vector space V, then $q(-\alpha) = -q(\alpha)$. (3)

Part II: Work out Problems.

1. Let V and W be vector spaces over a field K and let T: V $\to$ W be a mapping. State what it means to say T is linear transformation. (3)
2. Let T be the mapping $T : P_3 \to P_2$ defined by $T(a_0 + a_1x + a_2x^2 + a_3x^3) = 3a_0 + a_3x^2$. Then
   (a) show that T is linear. (12)
   (b) find a basis for the kernel of T. (7)
3. Let A and B be n $\times$ n similar matrices.Then prove that A and B have the same Characterstic polynomial. (11)
4. Find an orthonormal martix P for the symmetric matrix $A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 5 \end{pmatrix}$ such that P^TAP is a diagonal matrix. (26)
5. Consider the bases $\mathcal{B} = \{1 + x + x^2, x + x^2, x^2\}$ and $\mathcal{C} = \{1, x, x^2\}$ of P_2.
   (a) Find the cooordinate vector $[p(x)]_{\mathcal{B}}$ of p(x) where p(x)= $1 + x^2$. (6)
   (b)] Find the change of basis matrix $P_{\mathcal{C}\leftarrow\mathcal{B}}$ from $\mathcal{B}$ to $\mathcal{C}$. (5)
   (c) Use the results in (a) and (b) to compute $[p(x)]_{\mathcal{C}}$ where p(x)= $1 + x^2$. (4)

6. (a) Find the quadratic form $q\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ that corresponds to the symmetric matrix

$$A = \begin{pmatrix} 4 & 1 & -3 \\ 1 & 0 & 2 \\ -3 & 2 & 5 \end{pmatrix}.$$ (8)

(b) Find the symmetric matrix corresponding to the quadratic form $q(x_1, x_2, x_3) = 2x_1^2 + 2x_1x_2 + 4x_2x_3 - 10x_1x_3 - x_2^2$. (7)

END OF FIRST OPPORTUNITY EXAMINATION QUESTION PAPER