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QUALIFICATION : **BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS**	
QUALIFICATION CODE: **07BSAM**	LEVEL: **5**
COURSE: **LINEAR ALGEBRA 1**	COURSE CODE: **LIA502S**
DATE: **NOVEMBER 2024**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

FIRST OPPORTUNITY: EXAMINATION QUESTION PAPER

EXAMINER:	MR GABRIEL S MBOKOMA, MR ILENIKEMANYA NDADI
MODERATOR:	DR DAVID IIYAMBO

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

This paper consists of 3 pages including this front page.

Question 1

1.1 Consider the vectors $\mathbf{p} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{q} = \mathbf{i} - 3\mathbf{j} + 12\mathbf{k}$

a) Find the unit vector in the direction of $\mathbf{p}$. [3]

b) Find the angle (*in degrees*) between $\mathbf{p}$ and $\mathbf{q}$. Give you answer correct to 1 d.p. [8]

1.2 Find a unit vector perpendicular to both the vectors $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$. [5]

1.3 Prove that if $\mathbf{x}$ and $\mathbf{y}$ are orthogonal vectors in $\mathbb{R}^n$, then

$$\|\mathbf{x} + \mathbf{y}\|^2 = \|\mathbf{x}\|^2 + \|\mathbf{y}\|^2.$$

[6]

Question 2

2.1 Let $A = \left(a_{ij} \right)$ be an $n \times n$ matrix.

a) When do we say that A is a symmetric matrix? [2]

b) Prove that $A + A^T$ is a symmetric matrix. [5]

c) Prove that if A is an invertible symmetric matrix, then A^{-1} is also symmetric. [6]

2.2 Consider the following matrix.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos x & \sin x \\ 0 & \sin x & -\cos x \end{pmatrix}.$$

a) Use the *Cofactor expansion method* to evaluate the determinant of A through column one (1). [6]

b) Is A invertible? If it is, find A^{-1} using the adjoint matrix approach. [12]

Question 3

Given that matrix

$$B = \begin{pmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{pmatrix}$$

is symmetric, find the value of ab? [7]

Question 4

Use the *Gaussian elimination method* to find the solution of the following system of linear equations, if it exists.

$$\begin{aligned} x_1 + 3x_2 - x_3 &= 1 \\ 2x_1 + x_2 + x_3 &= 4 \\ 3x_1 + 4x_2 + 2x_3 &= -1 \end{aligned}$$

[14]

Question 5

a) Prove that a vector space cannot have more than one zero vector. [6]

b) Let M_{nn} be a vector space whose elements are all the $n \times n$ matrices, with the usual addition and scalar multiplication for matrices. Determine whether the following set is a subspace of M_{nn}.

$$S = \{A \in M_{nn} \,|\, tr(A) = 0\}$$

[11]

c) Prove or disprove that if S and T are subspaces of a vector space V, then $S \cap T$ is also a subspace of V. [9]
