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QUALIFICATION : **BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS**	
QUALIFICATION CODE: **07BSAM; 07BSOC**	LEVEL: **6**
COURSE: **LINEAR ALGEBRA 2**	COURSE CODE: **LIA601S**
DATE: **NOVEMBER 2024**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

FIRST OPPORTUNITY EXAMINATION: QUESTION PAPER

EXAMINER: DR. NEGA CHERE

MODERATOR: DR. DAVID IIYAMBO

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

ATTACHMENTS:

NONE

This paper consists of 3 pages including this front page.

QUESTION 1 [27]

Let $T: P_2 \rightarrow P_2$ be a mapping defined by:

$$T(a_0 + a_1x + a_2x^2) = a_0 + a_1(x+1) + a_2(x+1)^2.$$

1.1. Show that T is linear. [13]

1.2. Find the kernel of T and use it to determine whether T is singular or nonsingular. [8]

1.3. Show that the mapping $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(x,y,z) = (x \cdot y,\ x+y+z)$ is not linear. [6]

QUESTION 2 [20]

2.1. Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a mapping such that $T\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ and $T\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$. Then find $T\begin{bmatrix} a \\ b \end{bmatrix}$ and use it to determine $T\begin{bmatrix} 3 \\ 1 \end{bmatrix}$. [10]

2.2. Find the coordinate vector of the vector $v = (4, -2, 5)$ with respect to the ordered basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix} \right\}$ for $\mathbb{R}^3$. [10]

QUESTION 3 [8]

If A and B are n x n similar matrices then prove that A and B have the same characteristic polynomial. [8]

QUESTION 4 [11]

4.1. If λ is an eigenvalue of an invertible matrix A with corresponding eigenvector x, then show that λ^{-1} is an eigenvalue of A^{-1} with corresponding eigenvector x. [6]

4.2. Let A be a 2×2 matrix. Show that the characteristic polynomial $p(\lambda)$ of A is given by

$$p(\lambda) = \lambda^2 - tr(A)\lambda + \det(A).$$ [5]

QUESTION 5 [23]

Let $A = \begin{pmatrix} 2 & 1 & -2 \\ 2 & 3 & -4 \\ 1 & 1 & -1 \end{pmatrix}$.

5.1. Verify whether $\lambda = 1$ is an eigenvalue of A. If it is, find the corresponding eigenvector(s). [16]

5.2. Verify whether the vector $x = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ is an eigenvector for A. If it is, find the corresponding eigenvalue. [7]

QUESTION 6 [11]

Consider the following two bases of $\mathbb{R}^3$: $S = \{e_1, e_2, e_3\} = \{(1,0,0), (0,1,0), (0,0,1)\}$ and $E = \{v_1, v_2, v_3\} = \{(1,1,0), (0,1,1), (1,2,2)\}$.

6.1. Find the change of basis matrix from S to E, $P_{E \leftarrow S}$. [7]

6.2. Use the result in 6.1. to find $[v]_E$ where $v = (1, 3, -2)$. [4]

END OF FIRST OPPORTUNITY EXAMINATION QUESTION PAPER