



NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

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QUALIFICATION : Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSAM	LEVEL: 7
COURSE: DESIGN AND ANALYSIS OF EXPERIMENTS	COURSE CODE: DAE702S
DATE: JANUARY 2025	SESSION: 1
DURATION: 3 HOURS	MARKS: 100

SECOND OPPORTUNITY / SUPPLEMENTARY: EXAMINATION QUESTION PAPER

EXAMINER: *Dr D Ntirampeba, NUST*

MODERATOR: *Dr P Iiyambo, UNAM*

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the answer sheet. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Write all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

ATTACHEMENTS

1. F Table & t-Table

This paper consists of 4 pages excluding this front page

Question 1 [20 marks]

1.1 Briefly explain the following terminologies as they are applied to Experimental Design and Analysis of Variance.

- (i) Confounded Factors [2]
- (ii) Replicate runs [2]
- (iii) Effect [2]

1.2 Let the fixed effects model for one-way analysis of variance be

$$y_{ij} = \mu + \tau_i + \epsilon_{ij},$$

where y_{ij} is the j^{th} observation under the i^{th} treatment, μ is the overall mean, τ_i is the effect of the i^{th} treatment, and $\epsilon_{ij} \sim N(0, \sigma^2)$ is random error component, $i = 1, 2, \dots, k$ and $j = 1, 2, \dots, n_k$. Show that $\sum_{i=1}^k \tau_i = 0$ [5]

1.3 An industrial engineer wanted to determine the settings that would allow him to toss an object a given distance to fend off attackers or door-to-door salespeople. He varied three factors on the trebuchet (medieval missile launcher). A, the arm length 4 to 8 inches from the counterweight end to the point where the weights were hung; B, counterweight 10 to 20 pounds; and C, missile weight 2 to 3 ounces. A quadratic response model was fitted and the following results were obtained.

Call:

```
rsm(formula = y ~ SO(x1, x2, x3), data = Treb)
```

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	90.00000	1.16905	76.9859	7.006e-09	***
x1	19.75000	0.71589	27.5880	1.171e-06	***
x2	19.75000	0.71589	27.5880	1.171e-06	***
x3	-11.50000	0.71589	-16.0639	1.703e-05	***
x1:x2	-6.25000	1.01242	-6.1733	0.0016247	**
x1:x3	4.75000	1.01242	4.6917	0.0053768	**
x2:x3	6.75000	1.01242	6.6672	0.0011461	**
x1^2	-9.37500	1.05376	-8.8967	0.0002986	***
x2^2	-1.37500	1.05376	-1.3048	0.2487686	
x3^2	-3.37500	1.05376	-3.2028	0.0239200	*

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Multiple R-squared: 0.9975, Adjusted R-squared: 0.9929

F-statistic: 218.9 on 9 and 5 DF, p-value: 5.964e-06

Analysis of Variance Table

Response: y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
FO(x1, x2, x3)	3	7299.0	2433.00	593.4146	8.448e-07
TWI(x1, x2, x3)	3	428.8	142.92	34.8577	0.0008912
PQ(x1, x2, x3)	3	351.5	117.16	28.5759	0.0014236
Residuals	5	20.5	4.10		
Lack of fit	3	14.5	4.83	1.6111	0.4051312
Pure error	2	6.0	3.00		

Stationary point of response surface:

x1	x2	x3
0.9236846	-1.7161183	-2.7698217

Stationary point in original units:

A	B	C
7.847369	6.419409	1.115089

Eigenanalysis:

eigen() decomposition

\$values

[1] 1.280298 -3.551452 -11.853845

\$vectors

	[,1]	[,2]	[,3]
x1	-0.1236692	0.5238084	0.8428112
x2	0.8323200	-0.4077092	0.3755217
x3	0.5403233	0.7479291	-0.3855551

Based on these results,

- Write down the fitted quadratic model; [2]
- Comment on the adequacy of the fitted quadratic model; [3]
- What is the stationary point? [1]
- Characterize the stationary point (hint: is it a minimum, maximum or saddle point?). Justify your answer. [3]

Question 2 [20 marks]

The yield of a chemical process was measured using five batches of raw material, five acid concentrations, five standing times (A, B, C, D, E), and five catalyst concentrations ($\alpha, \beta, \gamma, \delta, \epsilon$). The Graeco-Latin square that follows was used.

Batch	Acid concentration				
	1	2	3	4	5
1	$A\alpha = 26$	$B\beta = 16$	$C\gamma = 19$	$D\delta = 16$	$E\epsilon = 13$
2	$B\gamma = 18$	$C\delta = 21$	$D\epsilon = 18$	$E\alpha = 11$	$A\beta = 21$
3	$C\epsilon = 20$	$D\alpha = 12$	$E\beta = 16$	$A\gamma = 25$	$B\gamma = 13$
4	$D\beta = 15$	$E\gamma = 15$	$A\delta = 22$	$B\epsilon = 14$	$C\alpha = 17$
5	$E\delta = 10$	$A\epsilon = 24$	$B\alpha = 17$	$C\beta = 17$	$D\delta = 14$

- 2.1 Write down an appropriate effects model for this design. [5]
- 2.2 Test the effects of standing times and catalyst concentrations on the yield of the chemical process (use $\alpha = 0.05$). [15]

Question 3 [40 marks]

Consider a study performed to determine if there is a difference in the strength of monofilament fibre produced by the three different machines. The data from this experiment are shown in the table below.

Machine 1		Machine 2		Machine 3	
y	x	y	x	y	x
36	20	40	22	35	21
41	25	48	28	37	23
39	24	39	22	42	26
42	25	45	30	34	21
49	32	44	28	32	15

- 3.1 Write down an appropriate model for this dataset. [3]
- 3.2 Use the analysis of covariance to remove the effect of the thickness (x) on strength (y) when testing for differences in strength between machines. Use 5% significance level. Clearly outline all the steps. [18]
- 3.3 Compute the estimate of the regression coefficient. [2]
- 3.4 Compute a 95 percent confidence interval on the adjusted mean strength of fibers produced by machine 1. [5]

3.5 Test for the significance of the regression coefficient. Use 5% significance level. Clearly outline all the steps. [4]

3.6 At 5% significance level, test the validity of the assumption that **the machines do not affect the the thickness**. [8]

Question 4 [20 marks]

An article in the Journal of Testing and Evaluation (Vol.16, no.2, pp.508-515) investigated the effects of cyclic loading and environmental conditions on fatigue crack growth at a constant 22 MPa stress for a particular material. The data from this experiment are shown in the table below (the response is crack growth rate).

Frequency	Environment		
	Air	H2O	Salt H2O
10	2.29	2.06	1.9
	2.47	2.05	1.93
	2.48	2.23	1.75
	2.12	2.03	2.06
1	2.65	3.2	3.1
	2.68	3.18	3.24
	2.06	3.96	3.98
	2.38	3.64	3.24
0.1	2.24	11	9.96
	2.71	11	10.01
	2.81	9.06	9.36
	2.08	11.3	10.4

4.1 Analyse the data to test the main effects of the frequency and environment. [15]

4.2 Is there a significant interaction between the frequency and environment? [5]

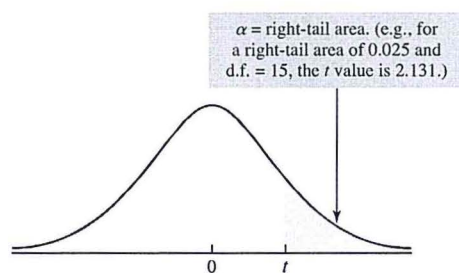
END OF QUESTION PAPER

CRITICAL VALUES for the "F" Distribution, ALPHA = .05.

Denominator DF	Numerator DF									
	1	2	3	4	5	6	7	8	9	10
1	161.448	199.500	215.707	224.583	230.162	233.986	236.768	238.883	240.543	241.882
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385	19.396
3	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.786
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.388	3.347
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.366	2.321
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320	2.275
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300	2.255
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.236
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265	2.220
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250	2.204
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236	2.190
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223	2.177
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211	2.165
31	4.160	3.305	2.911	2.679	2.523	2.409	2.323	2.255	2.199	2.153
32	4.149	3.295	2.901	2.668	2.512	2.399	2.313	2.244	2.189	2.142
33	4.139	3.285	2.892	2.659	2.503	2.389	2.303	2.235	2.179	2.133
34	4.130	3.276	2.883	2.650	2.494	2.380	2.294	2.225	2.170	2.123
35	4.121	3.267	2.874	2.641	2.485	2.372	2.285	2.217	2.161	2.114
36	4.113	3.259	2.866	2.634	2.477	2.364	2.277	2.209	2.153	2.106
37	4.105	3.252	2.859	2.626	2.470	2.356	2.270	2.201	2.145	2.098
38	4.098	3.245	2.852	2.619	2.463	2.349	2.262	2.194	2.138	2.091
39	4.091	3.238	2.845	2.612	2.456	2.342	2.255	2.187	2.131	2.084
40	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180	2.124	2.077
41	4.079	3.226	2.833	2.600	2.443	2.330	2.243	2.174	2.118	2.071
42	4.073	3.220	2.827	2.594	2.438	2.324	2.237	2.168	2.112	2.065
43	4.067	3.214	2.822	2.589	2.432	2.318	2.232	2.163	2.106	2.059
44	4.062	3.209	2.816	2.584	2.427	2.313	2.226	2.157	2.101	2.054
45	4.057	3.204	2.812	2.579	2.422	2.308	2.221	2.152	2.096	2.049
46	4.052	3.200	2.807	2.574	2.417	2.304	2.216	2.147	2.091	2.044
47	4.047	3.195	2.802	2.570	2.413	2.299	2.212	2.143	2.086	2.039
48	4.043	3.191	2.798	2.565	2.409	2.295	2.207	2.138	2.082	2.035
49	4.038	3.187	2.794	2.561	2.404	2.290	2.203	2.134	2.077	2.030
50	4.034	3.183	2.790	2.557	2.400	2.286	2.199	2.130	2.073	2.026

CRITICAL VALUES for the "F" Distribution, ALPHA = .05.

Denominator DF	Numerator DF									
	1	2	3	4	5	6	7	8	9	10
51	4.030	3.179	2.786	2.553	2.397	2.283	2.195	2.126	2.069	2.022
52	4.027	3.175	2.783	2.550	2.393	2.279	2.192	2.122	2.066	2.018
53	4.023	3.172	2.779	2.546	2.389	2.275	2.188	2.119	2.062	2.015
54	4.020	3.168	2.776	2.543	2.386	2.272	2.185	2.115	2.059	2.011
55	4.016	3.165	2.773	2.540	2.383	2.269	2.181	2.112	2.055	2.008
56	4.013	3.162	2.769	2.537	2.380	2.266	2.178	2.109	2.052	2.005
57	4.010	3.159	2.766	2.534	2.377	2.263	2.175	2.106	2.049	2.001
58	4.007	3.156	2.764	2.531	2.374	2.260	2.172	2.103	2.046	1.998
59	4.004	3.153	2.761	2.528	2.371	2.257	2.169	2.100	2.043	1.995
60	4.001	3.150	2.758	2.525	2.368	2.254	2.167	2.097	2.040	1.993
61	3.998	3.148	2.755	2.523	2.366	2.251	2.164	2.094	2.037	1.990
62	3.996	3.145	2.753	2.520	2.363	2.249	2.161	2.092	2.035	1.987
63	3.993	3.143	2.751	2.518	2.361	2.246	2.159	2.089	2.032	1.985
64	3.991	3.140	2.748	2.515	2.358	2.244	2.156	2.087	2.030	1.982
65	3.989	3.138	2.746	2.513	2.356	2.242	2.154	2.084	2.027	1.980
66	3.986	3.136	2.744	2.511	2.354	2.239	2.152	2.082	2.025	1.977
67	3.984	3.134	2.742	2.509	2.352	2.237	2.150	2.080	2.023	1.975
68	3.982	3.132	2.740	2.507	2.350	2.235	2.148	2.078	2.021	1.973
69	3.980	3.130	2.737	2.505	2.348	2.233	2.145	2.076	2.019	1.971
70	3.978	3.128	2.736	2.503	2.346	2.231	2.143	2.074	2.017	1.969
71	3.976	3.126	2.734	2.501	2.344	2.229	2.142	2.072	2.015	1.967
72	3.974	3.124	2.732	2.499	2.342	2.227	2.140	2.070	2.013	1.965
73	3.972	3.122	2.730	2.497	2.340	2.226	2.138	2.068	2.011	1.963
74	3.970	3.120	2.728	2.495	2.338	2.224	2.136	2.066	2.009	1.961
75	3.968	3.119	2.727	2.494	2.337	2.222	2.134	2.064	2.007	1.959
76	3.967	3.117	2.725	2.492	2.335	2.220	2.133	2.063	2.006	1.958
77	3.965	3.115	2.723	2.490	2.333	2.219	2.131	2.061	2.004	1.956
78	3.963	3.114	2.722	2.489	2.332	2.217	2.129	2.059	2.002	1.954
79	3.962	3.112	2.720	2.487	2.330	2.216	2.128	2.058	2.001	1.953
80	3.960	3.111	2.719	2.486	2.329	2.214	2.126	2.056	1.999	1.951
81	3.959	3.109	2.717	2.484	2.327	2.213	2.125	2.055	1.998	1.950
82	3.957	3.108	2.716	2.483	2.326	2.211	2.123	2.053	1.996	1.948
83	3.956	3.107	2.715	2.482	2.324	2.210	2.122	2.052	1.995	1.947
84	3.955	3.105	2.713	2.480	2.323	2.209	2.121	2.051	1.993	1.945
85	3.953	3.104	2.712	2.479	2.322	2.207	2.119	2.049	1.992	1.944
86	3.952	3.103	2.711	2.478	2.321	2.206	2.118	2.048	1.991	1.943
87	3.951	3.101	2.709	2.476	2.319	2.205	2.117	2.047	1.989	1.941
88	3.949	3.100	2.708	2.475	2.318	2.203	2.115	2.045	1.988	1.940
89	3.948	3.099	2.707	2.474	2.317	2.202	2.114	2.044	1.987	1.939
90	3.947	3.098	2.706	2.473	2.316	2.201	2.113	2.043	1.986	1.938
91	3.946	3.097	2.705	2.472	2.315	2.200	2.112	2.042	1.984	1.936
92	3.945	3.095	2.704	2.471	2.313	2.199	2.111	2.041	1.983	1.935
93	3.943	3.094	2.703	2.470	2.312	2.198	2.110	2.040	1.982	1.934
94	3.942	3.093	2.701	2.469	2.311	2.197	2.109	2.038	1.981	1.933
95	3.941	3.092	2.700	2.467	2.310	2.196	2.108	2.037	1.980	1.932
96	3.940	3.091	2.699	2.466	2.309	2.195	2.106	2.036	1.979	1.931
97	3.939	3.090	2.698	2.465	2.308	2.194	2.105	2.035	1.978	1.930
98	3.938	3.089	2.697	2.465	2.307	2.193	2.104	2.034	1.977	1.929
99	3.937	3.088	2.696	2.464	2.306	2.192	2.103	2.033	1.976	1.928
100	3.936	3.087	2.696	2.463	2.305	2.191	2.103	2.032	1.975	1.927



The t -Distribution

α :	0.10	0.05	0.025	0.01	0.005
d.f. = 1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
33	1.308	1.692	2.035	2.445	2.733
34	1.307	1.691	2.032	2.441	2.728
35	1.306	1.690	2.030	2.438	2.724
36	1.306	1.688	2.028	2.435	2.719
37	1.305	1.687	2.026	2.431	2.715
38	1.304	1.686	2.024	2.429	2.712
39	1.304	1.685	2.023	2.426	2.708
40	1.303	1.684	2.021	2.423	2.704
41	1.303	1.683	2.020	2.421	2.701
42	1.302	1.682	2.018	2.418	2.698
43	1.302	1.681	2.017	2.416	2.695
44	1.301	1.680	2.015	2.414	2.692
45	1.301	1.679	2.014	2.412	2.690