



**NAMIBIA UNIVERSITY  
OF SCIENCE AND TECHNOLOGY**

**FACULTY OF ENGINEERING AND SPATIAL SCIENCE  
DEPARTMENT OF MINING AND PROCESS ENGINEERING**

|   |                                   |
|---|-----------------------------------|
| <b>QUALIFICATION : BACHELORS OF ENGINEERING IN MINING ENGINEERING</b> |                                   |
| <b>QUALIFICATION CODE: BEMIN</b>                                      | <b>LEVEL: 6</b>                   |
| <b>COURSE CODE: GSS710S</b>   | <b>COURSE NAME: GEOSTATISTICS</b> |
| <b>SESSION: JUNE 2022</b>   | <b>PAPER: THEORY</b>              |
| <b>DURATION: 3 HOURS</b>  | <b>MARKS: 100</b>                 |

|                               |                                     |
|-------------------------------|-------------------------------------|
| <b>FIRST OPPORTUNITY EXAM</b> |                                     |
| <b>EXAMINER(S)</b>            | <b>Mallikarjun Rao Pillalamarry</b> |
| <b>MODERATOR:</b>             | <b>Lawrence Madziwa</b>             |

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| <b>INSTRUCTIONS</b>   |
| <ol style="list-style-type: none"><li>1. Answer all questions.</li><li>2. Read all the questions carefully before answering.</li><li>3. Marks for each question are indicated at the end of each question.</li><li>4. Please ensure that your writing is legible, neat and presentable.</li></ol> |

**PERMISSIBLE MATERIALS**

1. Examination paper.

**THIS QUESTION PAPER CONSISTS OF 10 PAGES (Including this front page)**

**This EXAM has two sections. Section A and B.**

**Time allowed: 3 hours**

**SECTION A**

**Instructions: Answer Question 1 and any 2 other questions. Excess questions will not be marked.**

**Question 1 is compulsory.**

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**Question 1**

- a) With illustrations, explain the differences between quantitative and qualitative data interpretation and evaluation. For a mining Engineer on an operating mine, how is qualitative data used to convey the positive or negative aspects of mining operations especially concerning the ore body? (8)
- b) State the advantages and disadvantages of each approach in interpreting qualitative and quantitative data interpretation. (4)
- c) Given the following data set:  
SET A: 4 3 7 5 3 2 4 5 6 2 6  
SET B: 2 2 4 4 5 5 6 6 7 3 3
- i) From a statistical and geostatistical point of view; discuss whether this data set is the same or different population. (2)
- ii) Calculate the means of Data set A and B. (4)
- iii) Find the statistical variance  $\{ \sigma^2 = [ \sum x_i - \mu ]^2 / (n-1) \}$  for both populations. Where  $\mu$  is the mean, and  $x_i$  are the individual values in data set. (4)
- iv) Find the Geostatistical variance  $\{ \sigma^2 = [ \sum x_i - x_{(i+h)} ]^2 / (n-1) \}$  where  $i$  and  $i+h$  denote next neighbour values in a data set. (4)
- v) Discuss the significance of these differences in the variance and what they imply in case of blocks of an ore body. (4)

- Question 2** State the three main stages required to move from exploration to mining an ore deposit. For each stage state the financial implications associated with that particular stage and the most probable source of funding. (10)

**Question 3**

- a) Discuss the revenue factors involved in operating a mining venture. (2)
- b) Discuss the main factors involved in the valuation of an ore body. (4)
- c) In an ore body, discuss the best way to deal with grade outliers (high grades and very low grades). (2)
- d) In the country of Vietnam, there is a correlation coefficient of 0.9 between GDP and Foreign Direct Investment (FDI). Explain this correlation coefficient and state what variables drive this economy to such levels? (2)

- Question 4** Discuss with examples the difference between measured, indicated and inferred reserves on a mine setting. (10)

## SECTION B

**Instructions: Answer Question 1 and any 2 other questions. Excess questions will not be marked.**

**Question 1 is compulsory.**

### Question 1

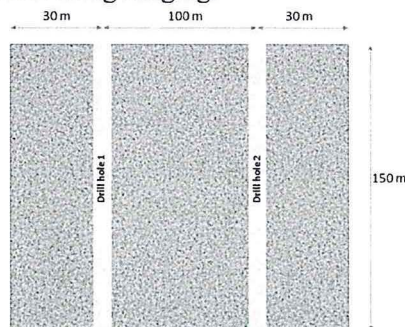
- a) How the sample mean and variance changes with support? (2)
- b) Why Kriging method of estimation is more reliable than other estimation methods? (1)
- c) What is the necessary condition for a variogram model to be used in Kriging estimation method? (1)
- d) What is auxiliary function  $F(l)$  gives? (1)
- e) What 'C' matrix in Kriging system of equations represents? (1)
- d) Why we have Lagrangian Multiplier in Kriging system of equations? (2)
- f) What is zonal anisotropy and what is reason for a variogram exhibiting zonal anisotropy? (2)

### Question 2

- a) The Zn grade for 2 m long core sections has modeled with a spherical variogram with a vertical range of 15 m, a sill of  $10(\%)^2$ . As per the mine planning the bench height will be 10 m and the data needs to be regularized over this height. Calculate the variogram values for 10 m, 15 m, and 20 m lags for 10 m core sections. [Assume that the point variogram parameters derived are appropriate] (10)
- b) Chip samples are collected from an underground tunnel at regularly spaced for every 5 m, and grades are available for 11 out of 13 samples as shown in Figure below. Calculate the experimental variogram for the first two lags for the given data. (10)

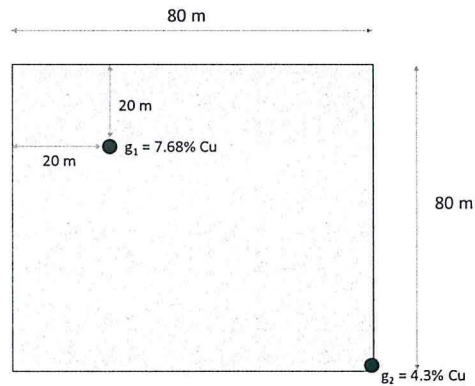


- Question 3 A 160 m x 150 m panel of Zn ore was intersected by two exploration drill holes as shown in Figure. The average Zn grade for the drill hole 1 and drill hole 2 for the intersected length is 4.5% and 3.4% respectively. Spatial continuity of Zinc grades in the orebody is following spherical variogram with a sill value of  $2.5 (\%)^2$  and range of 300 m. Estimate the grade of the panel using Kriging. (20)



$$C^{-1} = \begin{bmatrix} -0.65 & 0.65 & 0.5 \\ 0.65 & -0.65 & 0.5 \\ 0.5 & 0.5 & -0.99 \end{bmatrix}$$

**Question 4** Average grade of panel in a copper deposit to be estimated, and the panel was intersected two exploration drill holes as shown in Figure. The average grades at the intersected part of drill holes are given in the Figure as  $g_1$  and  $g_2$ . If the average grade  $g_1$  and  $g_2$  is used as the grade of the panel, estimate the extension variance. Spatial continuity of the grade in the deposit can be described using a spherical form with a range of influence of 90 m and a sill of  $0.6(\%)^2$ . (20)



## Standard Normal Probabilities

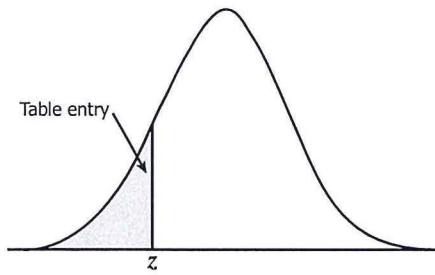


Table entry for  $z$  is the area under the standard normal curve to the left of  $z$ .

| $z$  | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| -3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| -1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| -0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| -0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| -0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| -0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| -0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| -0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| -0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| -0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| -0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| -0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |



Auxiliary function  $\gamma(L,B)$  for Spherical model with range 1.0 and sill 1.0

| L    | B     |       |       |       |       |       |       |       |       |       | L    | B     |       |       |       |       |       |       |       |       |       |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|      | .1    | .2    | .3    | .4    | .5    | .6    | .7    | .8    | .9    | 1.0   |      | 1.2   | 1.4   | 1.6   | 1.8   | 2.0   | 2.5   | 3.0   | 3.5   | 4.0   | 5.0   |
| .05  | .094  | .132  | .175  | .219  | .263  | .306  | .348  | .388  | .426  | .461  | .05  | .524  | .575  | .617  | .652  | .681  | .737  | .777  | .806  | .828  | .861  |
| .10  | .161  | .188  | .223  | .261  | .300  | .340  | .379  | .416  | .452  | .486  | .10  | .545  | .594  | .634  | .667  | .695  | .748  | .786  | .814  | .836  | .867  |
| .15  | .231  | .252  | .280  | .312  | .347  | .383  | .419  | .453  | .486  | .518  | .15  | .573  | .619  | .656  | .687  | .714  | .764  | .799  | .825  | .846  | .875  |
| .20  | .302  | .318  | .341  | .369  | .400  | .432  | .464  | .495  | .526  | .555  | .20  | .605  | .648  | .682  | .711  | .735  | .782  | .814  | .838  | .857  | .884  |
| .25  | .372  | .385  | .404  | .428  | .455  | .483  | .512  | .541  | .568  | .594  | .25  | .641  | .679  | .711  | .737  | .759  | .801  | .831  | .853  | .870  | .894  |
| .30  | .440  | .451  | .467  | .488  | .511  | .536  | .562  | .588  | .613  | .636  | .30  | .678  | .712  | .741  | .764  | .784  | .822  | .848  | .868  | .883  | .905  |
| .35  | .507  | .516  | .529  | .547  | .568  | .590  | .612  | .635  | .657  | .678  | .35  | .715  | .746  | .771  | .792  | .809  | .843  | .866  | .884  | .897  | .917  |
| .40  | .571  | .578  | .590  | .605  | .623  | .642  | .662  | .683  | .702  | .721  | .40  | .753  | .780  | .801  | .820  | .835  | .864  | .884  | .899  | .911  | .928  |
| .45  | .632  | .638  | .648  | .661  | .677  | .693  | .711  | .729  | .746  | .762  | .45  | .790  | .812  | .831  | .847  | .860  | .884  | .902  | .915  | .924  | .939  |
| .50  | .689  | .695  | .703  | .715  | .728  | .742  | .758  | .773  | .787  | .801  | .50  | .825  | .844  | .860  | .872  | .883  | .904  | .918  | .929  | .937  | .949  |
| .55  | .743  | .748  | .755  | .765  | .776  | .789  | .802  | .814  | .827  | .838  | .55  | .858  | .873  | .886  | .897  | .906  | .922  | .934  | .943  | .949  | .959  |
| .60  | .793  | .797  | .803  | .811  | .821  | .831  | .842  | .853  | .863  | .872  | .60  | .888  | .901  | .911  | .919  | .926  | .939  | .948  | .955  | .960  | .968  |
| .65  | .839  | .842  | .847  | .854  | .862  | .870  | .879  | .888  | .896  | .903  | .65  | .915  | .925  | .933  | .939  | .944  | .954  | .961  | .966  | .970  | .976  |
| .70  | .879  | .882  | .886  | .892  | .898  | .905  | .912  | .919  | .925  | .930  | .70  | .939  | .946  | .952  | .956  | .960  | .967  | .972  | .976  | .979  | .983  |
| .75  | .915  | .917  | .920  | .925  | .930  | .935  | .940  | .945  | .949  | .953  | .75  | .959  | .964  | .968  | .971  | .974  | .978  | .982  | .984  | .986  | .989  |
| .80  | .945  | .946  | .949  | .952  | .956  | .960  | .963  | .966  | .969  | .971  | .80  | .975  | .978  | .981  | .983  | .984  | .987  | .989  | .991  | .992  | .993  |
| .85  | .968  | .970  | .971  | .974  | .976  | .978  | .981  | .982  | .984  | .985  | .85  | .987  | .989  | .990  | .991  | .992  | .993  | .994  | .995  | .996  | .997  |
| .90  | .986  | .987  | .988  | .989  | .990  | .991  | .992  | .993  | .994  | .994  | .90  | .995  | .996  | .996  | .997  | .997  | .997  | .998  | .998  | .998  | .999  |
| .95  | .996  | .997  | .997  | .998  | .998  | .998  | .998  | .999  | .999  | .999  | .95  | .999  | .999  | .999  | .999  | .999  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 1.00 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.00 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

Auxiliary function  $F(L; B)$  for Spherical model with range 1.0 and sill 1.0

| L    | B    |      |      |      |      |      |      |      |      |      | L    | B    |      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
|      | .1   | .2   | .3   | .4   | .5   | .6   | .7   | .8   | .9   | 1.0  |      | 1.2  | 1.4  | 1.6  | 1.8  | 2.0  | 2.5  | 3.0  | 3.5  | 4.0  | 5.0  |
| .10  | .078 | .120 | .165 | .211 | .256 | .300 | .342 | .383 | .422 | .457 | .10  | .520 | .572 | .614 | .650 | .679 | .735 | .775 | .804 | .827 | .860 |
| .20  | .120 | .155 | .196 | .237 | .280 | .321 | .362 | .401 | .438 | .473 | .20  | .534 | .584 | .625 | .659 | .688 | .743 | .781 | .810 | .832 | .864 |
| .30  | .165 | .196 | .231 | .270 | .309 | .349 | .387 | .424 | .460 | .493 | .30  | .551 | .600 | .639 | .672 | .700 | .752 | .789 | .817 | .838 | .869 |
| .40  | .211 | .237 | .270 | .305 | .342 | .379 | .415 | .451 | .484 | .516 | .40  | .572 | .618 | .655 | .687 | .713 | .763 | .799 | .825 | .845 | .874 |
| .50  | .256 | .280 | .309 | .342 | .376 | .411 | .445 | .479 | .511 | .541 | .50  | .593 | .637 | .673 | .703 | .728 | .775 | .809 | .834 | .853 | .881 |
| .60  | .300 | .321 | .349 | .379 | .411 | .443 | .476 | .507 | .538 | .566 | .60  | .616 | .657 | .691 | .719 | .743 | .788 | .820 | .843 | .861 | .887 |
| .70  | .342 | .362 | .387 | .415 | .445 | .476 | .506 | .536 | .565 | .591 | .70  | .638 | .677 | .709 | .736 | .758 | .800 | .830 | .852 | .870 | .894 |
| .80  | .383 | .401 | .424 | .451 | .479 | .507 | .536 | .564 | .591 | .616 | .80  | .660 | .697 | .727 | .752 | .773 | .813 | .841 | .861 | .878 | .901 |
| .90  | .422 | .438 | .460 | .484 | .511 | .538 | .565 | .591 | .616 | .640 | .90  | .682 | .716 | .744 | .767 | .787 | .824 | .851 | .870 | .885 | .907 |
| 1.00 | .457 | .473 | .493 | .516 | .541 | .566 | .591 | .616 | .640 | .662 | 1.00 | .701 | .733 | .760 | .782 | .800 | .835 | .860 | .878 | .892 | .913 |
| 1.20 | .520 | .534 | .551 | .572 | .593 | .616 | .638 | .660 | .682 | .701 | 1.20 | .736 | .764 | .788 | .807 | .823 | .854 | .876 | .892 | .905 | .923 |
| 1.40 | .572 | .584 | .600 | .618 | .637 | .657 | .677 | .697 | .716 | .733 | 1.40 | .764 | .790 | .811 | .828 | .842 | .870 | .890 | .904 | .915 | .931 |
| 1.60 | .614 | .625 | .639 | .655 | .673 | .691 | .709 | .727 | .744 | .760 | 1.60 | .788 | .811 | .829 | .845 | .858 | .883 | .901 | .914 | .924 | .938 |
| 1.80 | .650 | .659 | .672 | .687 | .703 | .719 | .736 | .752 | .767 | .782 | 1.80 | .807 | .828 | .845 | .859 | .871 | .894 | .910 | .921 | .931 | .944 |
| 2.00 | .679 | .688 | .700 | .713 | .728 | .743 | .758 | .773 | .787 | .800 | 2.00 | .823 | .842 | .858 | .871 | .882 | .903 | .917 | .928 | .936 | .948 |
| 2.50 | .735 | .743 | .752 | .763 | .775 | .788 | .800 | .813 | .824 | .835 | 2.50 | .854 | .870 | .883 | .894 | .903 | .920 | .932 | .941 | .948 | .957 |
| 3.00 | .775 | .781 | .789 | .799 | .809 | .820 | .830 | .841 | .851 | .860 | 3.00 | .876 | .890 | .901 | .910 | .917 | .932 | .942 | .950 | .955 | .964 |
| 3.50 | .804 | .810 | .817 | .825 | .834 | .843 | .852 | .861 | .870 | .878 | 3.50 | .892 | .904 | .914 | .921 | .928 | .941 | .950 | .956 | .961 | .969 |
| 4.00 | .827 | .832 | .838 | .845 | .853 | .861 | .870 | .878 | .885 | .892 | 4.00 | .905 | .915 | .924 | .931 | .936 | .948 | .955 | .961 | .966 | .972 |
| 5.00 | .860 | .864 | .869 | .874 | .881 | .887 | .894 | .901 | .907 | .913 | 5.00 | .923 | .931 | .938 | .944 | .948 | .957 | .964 | .969 | .972 | .977 |

Auxiliary function H(L,B) for Spherical model with range 1.0 and sill 1.0

| L    | B    |      |      |      |      |      |      |      |      |      | L    | B    |      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
|      | .1   | .2   | .3   | .4   | .5   | .6   | .7   | .8   | .9   | .10  |      | 1.2  | 1.4  | 1.6  | 1.8  | 2.0  | 2.5  | 3.0  | 3.5  | 4.0  | 5.0  |
| .10  | .114 | .177 | .243 | .310 | .374 | .436 | .494 | .546 | .593 | .633 | .10  | .694 | .738 | .771 | .796 | .817 | .853 | .878 | .895 | .908 | .927 |
| .20  | .177 | .227 | .285 | .346 | .406 | .464 | .518 | .568 | .613 | .651 | .20  | .709 | .751 | .782 | .806 | .826 | .860 | .884 | .900 | .913 | .930 |
| .30  | .243 | .285 | .336 | .390 | .445 | .499 | .550 | .597 | .639 | .674 | .30  | .729 | .767 | .797 | .819 | .837 | .870 | .891 | .907 | .919 | .935 |
| .40  | .310 | .346 | .390 | .439 | .489 | .539 | .586 | .629 | .668 | .701 | .40  | .751 | .786 | .813 | .834 | .850 | .880 | .900 | .914 | .925 | .940 |
| .50  | .374 | .406 | .445 | .489 | .535 | .580 | .623 | .663 | .698 | .728 | .50  | .774 | .806 | .830 | .849 | .864 | .891 | .909 | .922 | .932 | .946 |
| .60  | .436 | .464 | .499 | .539 | .580 | .621 | .660 | .697 | .728 | .755 | .60  | .796 | .825 | .847 | .864 | .878 | .902 | .918 | .930 | .939 | .951 |
| .70  | .494 | .518 | .550 | .586 | .623 | .660 | .696 | .729 | .757 | .781 | .70  | .818 | .844 | .863 | .879 | .891 | .913 | .927 | .938 | .945 | .956 |
| .80  | .546 | .568 | .597 | .629 | .663 | .697 | .729 | .758 | .783 | .805 | .80  | .837 | .861 | .878 | .892 | .902 | .922 | .935 | .944 | .951 | .961 |
| .90  | .593 | .613 | .639 | .668 | .698 | .728 | .757 | .783 | .806 | .826 | .90  | .855 | .875 | .891 | .903 | .913 | .930 | .942 | .950 | .956 | .965 |
| 1.00 | .633 | .651 | .674 | .701 | .728 | .755 | .781 | .805 | .826 | .843 | 1.00 | .869 | .888 | .902 | .913 | .921 | .937 | .948 | .955 | .961 | .969 |
| 1.20 | .694 | .709 | .729 | .751 | .774 | .796 | .818 | .837 | .855 | .869 | 1.20 | .891 | .907 | .918 | .927 | .935 | .948 | .956 | .963 | .967 | .974 |
| 1.40 | .738 | .751 | .767 | .786 | .806 | .825 | .844 | .861 | .875 | .888 | 1.40 | .907 | .920 | .930 | .938 | .944 | .955 | .963 | .968 | .972 | .978 |
| 1.60 | .771 | .782 | .797 | .813 | .830 | .847 | .863 | .878 | .891 | .902 | 1.60 | .918 | .930 | .939 | .945 | .951 | .961 | .967 | .972 | .975 | .980 |
| 1.80 | .796 | .806 | .819 | .834 | .849 | .864 | .879 | .892 | .903 | .913 | 1.80 | .927 | .938 | .945 | .952 | .956 | .965 | .971 | .975 | .978 | .983 |
| 2.00 | .817 | .826 | .837 | .850 | .864 | .878 | .891 | .902 | .913 | .921 | 2.00 | .935 | .944 | .951 | .956 | .961 | .969 | .974 | .978 | .980 | .984 |
| 2.50 | .853 | .860 | .870 | .880 | .891 | .902 | .913 | .922 | .930 | .937 | 2.50 | .948 | .955 | .961 | .965 | .969 | .975 | .979 | .982 | .984 | .987 |
| 3.00 | .878 | .884 | .891 | .900 | .909 | .918 | .927 | .935 | .942 | .948 | 3.00 | .956 | .963 | .967 | .971 | .974 | .979 | .983 | .985 | .987 | .990 |
| 3.50 | .895 | .900 | .907 | .914 | .922 | .930 | .938 | .944 | .950 | .955 | 3.50 | .963 | .968 | .972 | .975 | .978 | .982 | .985 | .987 | .989 | .991 |
| 4.00 | .908 | .913 | .919 | .925 | .932 | .939 | .945 | .951 | .956 | .961 | 4.00 | .967 | .972 | .975 | .978 | .980 | .984 | .987 | .989 | .990 | .992 |
| 5.00 | .927 | .930 | .935 | .940 | .946 | .951 | .956 | .961 | .965 | .969 | 5.00 | .974 | .978 | .980 | .983 | .984 | .987 | .990 | .991 | .992 | .994 |

Auxiliary function F(L; L; B) for Spherical model with range 1.0 and sill 1.0

| L    | B    |      |      |      |      |      |      |      |      |      | L    | B    |      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
|      | .1   | .2   | .3   | .4   | .5   | .6   | .7   | .8   | .9   | 1.0  |      | 1.2  | 1.4  | 1.6  | 1.8  | 2.0  | 2.5  | 3.0  | 3.5  | 4.0  | 5.0  |
| .10  | .099 | .136 | .178 | .222 | .266 | .309 | .350 | .390 | .428 | .464 | .10  | .526 | .577 | .619 | .653 | .683 | .738 | .777 | .807 | .829 | .861 |
| .20  | .168 | .196 | .231 | .269 | .308 | .347 | .385 | .423 | .458 | .491 | .20  | .550 | .598 | .638 | .671 | .699 | .751 | .789 | .816 | .837 | .868 |
| .30  | .239 | .262 | .291 | .324 | .358 | .394 | .429 | .463 | .496 | .527 | .30  | .581 | .626 | .663 | .693 | .719 | .768 | .803 | .829 | .849 | .877 |
| .40  | .311 | .329 | .353 | .382 | .413 | .445 | .476 | .508 | .538 | .566 | .40  | .616 | .657 | .691 | .719 | .742 | .787 | .819 | .843 | .861 | .887 |
| .50  | .380 | .395 | .416 | .441 | .468 | .497 | .526 | .554 | .581 | .607 | .50  | .652 | .689 | .720 | .745 | .767 | .808 | .836 | .858 | .874 | .898 |
| .60  | .445 | .459 | .477 | .499 | .523 | .549 | .574 | .600 | .624 | .648 | .60  | .688 | .722 | .749 | .772 | .791 | .828 | .854 | .873 | .887 | .909 |
| .70  | .507 | .519 | .535 | .554 | .576 | .598 | .622 | .644 | .666 | .687 | .70  | .723 | .753 | .777 | .798 | .815 | .847 | .870 | .887 | .900 | .919 |
| .80  | .565 | .574 | .588 | .606 | .625 | .645 | .666 | .686 | .705 | .724 | .80  | .756 | .782 | .804 | .822 | .837 | .865 | .886 | .901 | .912 | .929 |
| .90  | .616 | .625 | .637 | .652 | .669 | .687 | .706 | .724 | .741 | .757 | .90  | .785 | .809 | .828 | .843 | .857 | .882 | .900 | .913 | .923 | .937 |
| 1.00 | .662 | .669 | .680 | .694 | .709 | .725 | .741 | .757 | .772 | .786 | 1.00 | .811 | .832 | .849 | .862 | .874 | .896 | .912 | .923 | .932 | .945 |
| 1.20 | .735 | .741 | .750 | .760 | .772 | .785 | .797 | .810 | .822 | .833 | 1.20 | .853 | .869 | .882 | .893 | .902 | .919 | .931 | .940 | .947 | .957 |
| 1.40 | .789 | .794 | .800 | .809 | .818 | .828 | .839 | .849 | .858 | .867 | 1.40 | .883 | .896 | .906 | .915 | .922 | .936 | .945 | .952 | .958 | .966 |
| 1.60 | .828 | .832 | .838 | .845 | .852 | .861 | .869 | .877 | .885 | .892 | 1.60 | .905 | .915 | .924 | .931 | .937 | .948 | .956 | .961 | .966 | .972 |
| 1.80 | .858 | .861 | .866 | .872 | .878 | .885 | .892 | .899 | .905 | .911 | 1.80 | .922 | .930 | .937 | .943 | .948 | .957 | .963 | .968 | .972 | .977 |
| 2.00 | .880 | .883 | .887 | .892 | .897 | .903 | .909 | .915 | .920 | .925 | 2.00 | .934 | .941 | .947 | .952 | .956 | .964 | .969 | .973 | .976 | .981 |
| 2.50 | .918 | .920 | .923 | .926 | .930 | .934 | .938 | .942 | .946 | .949 | 2.50 | .955 | .960 | .964 | .967 | .970 | .975 | .979 | .982 | .984 | .987 |
| 3.00 | .940 | .941 | .944 | .946 | .949 | .952 | .955 | .958 | .960 | .963 | 3.00 | .967 | .971 | .974 | .976 | .978 | .982 | .985 | .987 | .988 | .991 |
| 3.50 | .954 | .955 | .957 | .959 | .961 | .963 | .966 | .968 | .970 | .972 | 3.50 | .975 | .978 | .980 | .982 | .983 | .986 | .988 | .990 | .991 | .993 |
| 4.00 | .963 | .964 | .965 | .967 | .969 | .970 | .972 | .974 | .976 | .977 | 4.00 | .980 | .982 | .984 | .986 | .987 | .989 | .991 | .992 | .993 | .994 |
| 5.00 | .974 | .975 | .976 | .978 | .979 | .980 | .981 | .983 | .984 | .985 | 5.00 | .987 | .988 | .989 | .990 | .991 | .993 | .994 | .995 | .995 | .996 |



Auxiliary function  $\chi(L;B)$  for Spherical model with range 1.0 and sill 1.0

| <i>L</i> | <i>B</i> |      |      |      |      |      |      |      |      |      | <i>L</i> | <i>B</i> |      |      |      |      |      |      |      |      |      |
|----------|----------|------|------|------|------|------|------|------|------|------|----------|----------|------|------|------|------|------|------|------|------|------|
|          | .1       | .2   | .3   | .4   | .5   | .6   | .7   | .8   | .9   | 1.0  |          | 1.2      | 1.4  | 1.6  | 1.8  | 2.0  | 2.5  | 3.0  | 3.5  | 4.0  | 5.0  |
| .10      | .098     | .136 | .178 | .222 | .266 | .309 | .350 | .390 | .428 | .464 | .10      | .526     | .577 | .619 | .653 | .683 | .738 | .777 | .807 | .829 | .861 |
| .20      | .164     | .194 | .229 | .268 | .307 | .346 | .385 | .422 | .458 | .491 | .20      | .550     | .598 | .638 | .671 | .698 | .751 | .788 | .816 | .837 | .868 |
| .30      | .233     | .257 | .288 | .321 | .356 | .392 | .427 | .462 | .495 | .526 | .30      | .580     | .625 | .662 | .693 | .719 | .768 | .803 | .828 | .848 | .877 |
| .40      | .302     | .322 | .348 | .378 | .409 | .441 | .474 | .505 | .535 | .564 | .40      | .614     | .655 | .689 | .718 | .741 | .787 | .819 | .842 | .861 | .887 |
| .50      | .368     | .385 | .408 | .434 | .462 | .492 | .521 | .550 | .577 | .603 | .50      | .649     | .687 | .718 | .743 | .765 | .806 | .835 | .857 | .873 | .897 |
| .60      | .430     | .445 | .466 | .489 | .515 | .541 | .568 | .594 | .619 | .642 | .60      | .684     | .718 | .746 | .769 | .788 | .825 | .852 | .871 | .886 | .907 |
| .70      | .488     | .502 | .520 | .541 | .564 | .588 | .612 | .636 | .658 | .680 | .70      | .717     | .747 | .772 | .793 | .811 | .844 | .867 | .885 | .898 | .917 |
| .80      | .542     | .554 | .570 | .589 | .610 | .631 | .653 | .674 | .695 | .714 | .80      | .747     | .774 | .797 | .815 | .831 | .861 | .881 | .897 | .909 | .926 |
| .90      | .589     | .600 | .614 | .632 | .650 | .670 | .689 | .708 | .727 | .744 | .90      | .774     | .798 | .818 | .835 | .849 | .875 | .894 | .908 | .919 | .934 |
| 1.00     | .629     | .639 | .653 | .668 | .685 | .703 | .720 | .737 | .754 | .769 | 1.00     | .796     | .818 | .836 | .851 | .864 | .888 | .905 | .917 | .927 | .941 |
| 1.20     | .691     | .699 | .711 | .723 | .737 | .752 | .767 | .781 | .795 | .808 | 1.20     | .830     | .848 | .864 | .876 | .886 | .906 | .920 | .931 | .939 | .950 |
| 1.40     | .735     | .742 | .752 | .763 | .775 | .788 | .800 | .812 | .824 | .835 | 1.40     | .854     | .870 | .883 | .894 | .903 | .920 | .932 | .941 | .948 | .958 |
| 1.60     | .768     | .775 | .783 | .793 | .803 | .814 | .825 | .836 | .846 | .856 | 1.60     | .873     | .886 | .898 | .907 | .915 | .930 | .940 | .948 | .954 | .963 |
| 1.80     | .794     | .800 | .807 | .816 | .825 | .835 | .845 | .854 | .863 | .872 | 1.80     | .887     | .899 | .909 | .917 | .924 | .938 | .947 | .954 | .959 | .967 |
| 2.00     | .815     | .820 | .826 | .834 | .842 | .851 | .860 | .869 | .877 | .885 | 2.00     | .898     | .909 | .918 | .926 | .932 | .944 | .952 | .959 | .963 | .970 |
| 2.50     | .852     | .856 | .861 | .867 | .874 | .881 | .888 | .895 | .902 | .908 | 2.50     | .918     | .927 | .934 | .940 | .946 | .955 | .962 | .967 | .971 | .976 |
| 3.00     | .876     | .880 | .884 | .889 | .895 | .901 | .907 | .912 | .918 | .923 | 3.00     | .932     | .939 | .945 | .950 | .955 | .963 | .968 | .972 | .976 | .980 |
| 3.50     | .894     | .897 | .901 | .905 | .910 | .915 | .920 | .925 | .930 | .934 | 3.50     | .942     | .948 | .953 | .957 | .961 | .968 | .973 | .976 | .979 | .983 |
| 4.00     | .907     | .910 | .913 | .917 | .921 | .926 | .930 | .934 | .938 | .942 | 4.00     | .949     | .955 | .959 | .963 | .966 | .972 | .976 | .979 | .982 | .985 |
| 5.00     | .926     | .928 | .931 | .934 | .937 | .941 | .944 | .947 | .951 | .954 | 5.00     | .959     | .964 | .967 | .970 | .973 | .978 | .981 | .983 | .985 | .988 |

Regularised semi-variogram  $\gamma(h)$  for Spherical model with range *a* and sill 1.0 for various distances *h*

| <i>a/L</i> | <i>h/L</i> |      |      |      |      |      |      |      |      |      |
|------------|------------|------|------|------|------|------|------|------|------|------|
|            | 1.0        | 2.0  | 3.0  | 4.0  | 5.0  | 6.0  | 7.0  | 8.0  | 9.0  | 10.0 |
| .50        | .300       | .325 | .325 | .325 | .325 | .325 | .325 | .325 | .325 | .325 |
| 1.00       | .450       | .550 | .550 | .550 | .550 | .550 | .550 | .550 | .550 | .550 |
| 1.50       | .463       | .678 | .681 | .681 | .681 | .681 | .681 | .681 | .681 | .681 |
| 2.00       | .412       | .728 | .756 | .756 | .756 | .756 | .756 | .756 | .756 | .756 |
| 2.50       | .355       | .717 | .802 | .803 | .803 | .803 | .803 | .803 | .803 | .803 |
| 3.00       | .307       | .669 | .822 | .835 | .835 | .835 | .835 | .835 | .835 | .835 |
| 3.50       | .269       | .610 | .812 | .858 | .858 | .858 | .858 | .858 | .858 | .858 |
| 4.00       | .239       | .555 | .778 | .868 | .876 | .876 | .876 | .876 | .876 | .876 |
| 4.50       | .215       | .507 | .733 | .861 | .889 | .889 | .889 | .889 | .889 | .889 |
| 5.00       | .194       | .464 | .686 | .836 | .896 | .900 | .900 | .900 | .900 | .900 |
| 5.50       | .178       | .428 | .642 | .802 | .890 | .909 | .909 | .909 | .909 | .909 |
| 6.00       | .163       | .396 | .601 | .764 | .872 | .914 | .917 | .917 | .917 | .917 |
| 6.50       | .151       | .368 | .564 | .726 | .845 | .909 | .923 | .923 | .923 | .923 |
| 7.00       | .141       | .344 | .530 | .690 | .814 | .895 | .926 | .929 | .929 | .929 |
| 7.50       | .132       | .323 | .500 | .655 | .782 | .874 | .923 | .933 | .933 | .933 |
| 8.00       | .124       | .304 | .472 | .623 | .751 | .849 | .912 | .936 | .938 | .938 |
| 8.50       | .117       | .287 | .447 | .593 | .720 | .822 | .894 | .933 | .941 | .941 |
| 9.00       | .110       | .272 | .425 | .566 | .690 | .794 | .874 | .924 | .943 | .945 |
| 9.50       | .104       | .258 | .404 | .541 | .663 | .767 | .851 | .910 | .941 | .947 |
| 10.00      | .099       | .246 | .386 | .517 | .636 | .741 | .827 | .892 | .933 | .949 |

**Additional Information (GSS710S)**

Spherical Variogram Model

$$\gamma(h) = C \left[ \frac{3}{2} \left( \frac{h}{a} \right) - \frac{1}{2} \left( \frac{h}{a} \right)^3 \right] \quad \text{for } h < a$$

$$= C \quad \text{for } h \geq a$$

Relationship between sill of regularized and point variogram

$$C_L = C \left[ 1 - \frac{L}{2a} + \frac{L^3}{20a^3} \right] \quad \text{for } L < a$$

$$C_L = \frac{Ca}{L} \left[ \frac{15}{20} - \frac{4}{20} \frac{a}{L} \right] \quad \text{for } L \geq a$$

Auxiliary functions for Spherical variogram

$$\chi(l) = \frac{Cl}{8a} \left( 6 - \frac{l^2}{a^2} \right) \quad \text{when } l \leq a$$

$$= \frac{C}{8} \left( 8 - 3 \frac{a}{l} \right) \quad \text{when } l > a$$

$$F(l) = \frac{Cl}{20a} \left( 10 - \frac{l^2}{a^2} \right) \quad \text{when } l \leq a$$

$$= \frac{C}{20} \left( 20 - 15 \frac{a}{l} + 4 \frac{a^2}{l^2} \right) \quad \text{when } l > a$$

