



**PAMIBIA UNIVERSITY**  
OF SCIENCE AND TECHNOLOGY  
**FACULTY OF HEALTH AND APPLIED SCIENCES**

**DEPARTMENT OF MATHEMATICS AND STATISTICS**

<b>QUALIFICATION:</b> Bachelor of science in Applied Mathematics and Statistics	
<b>QUALIFICATION CODE:</b> 07BAMS	<b>LEVEL:</b> 6
<b>COURSE CODE:</b> PBT602S	<b>COURSE NAME:</b> PROBABILITY THEORY 2
<b>SESSION:</b> JANUARY 2020	<b>PAPER:</b> THEORY
<b>DURATION:</b> 3 HOURS	<b>MARKS:</b> 100

<b>SECOND OPPORTUNITY/SUPPLEMENTARY EXAMINATION QUESTION PAPER</b>	
<b>EXAMINER</b>	Dr. D. NTIRAMPEBA
<b>MODERATOR:</b>	Dr. D. B. GEMECHU

<b>INSTRUCTIONS</b>
<ol style="list-style-type: none"><li>1. Answer ALL the questions in the booklet provided.</li><li>2. Show clearly all the steps used in the calculations.</li><li>3. All written work must be done in blue or black ink and sketches must be done in pencil. Marks will not be awarded for answers obtained without showing the necessary steps leading to them</li></ol>

**PERMISSIBLE MATERIALS**

1. Non-programmable calculator without a cover.

**ATTACHMENTS**

1. None

**THIS QUESTION PAPER CONSISTS OF 2 PAGES** (Excluding this front page)

Question 1 [20 marks]

- 1.1 Briefly explain the following terminologies as they are applied to probability theory:
- (a) Boolean algebra  $\mathcal{B}(S)$  [2]
  - (b)  $\sigma$  algebra [3]
  - (c) Measure on a  $\mathcal{B}(S)$  algebra [3]
  - (d) Convolution of two integrable real-valued functions  $f$  and  $g$  [3]
- 1.2 Let  $S = \{a, b, c, d\}$ . Find:
- (a)  $\mathcal{P}(S)$ , [4]
  - (b) size of  $\mathcal{P}(S)$ . [1]
- 1.3 Consider the random variables  $X$  and  $Y$  that represent the number of vehicles that arrive at two separate corners during a certain 2-minute period. These two street corners are fairly close together so that it is important that the traffic engineers deal with them jointly if necessary. The joint distribution of  $X$  and  $Y$  is known to be

$$f(x, y) = \begin{cases} \frac{9}{16} \frac{1}{4^{x+y}} & , x = 0, 1, 2, \dots, y = 0, 1, 2, \dots, \\ 0 & , \text{otherwise} \end{cases}$$

Find the probability that less than 4 vehicles arrive at the two street corners during the stated time period. [4]

Question 2 [30 marks]

- 2.1 Let  $X$  and  $Y$  denote the lengths of life, in years, of two components in an electronic system. If the joint density function of these variables is

$$f(x, y) = \begin{cases} e^{-(x+y)} & , x > 0, y > 0, \\ 0 & , \text{otherwise,} \end{cases}$$

then find the mean value of  $Y$ . [6]

- 2.2 Suppose  $X$  and  $Y$  are random variables such that  $(X, Y)$  must belong to the rectangle in  $xy$ - plane containing all points  $(x, y)$  for which  $0 \leq x \leq 3$  and  $0 \leq y \leq 4$ . Suppose that the joint cumulative distribution of  $X$  and  $Y$  at any point  $(x, y)$  in this rectangle is specified as follows:  $F(x, y) = \frac{xy(x^2+y)}{156}$ .

- (a) Use the joint cumulative distribution,  $F(x, y)$ , to find  $(P(1 \leq x \leq 2, 1 \leq y \leq 2))$  [7]
- (b) Find the joint probability density function of  $X$  and  $Y$  [4]

- 2.3 Let  $X$  be a **discrete** random variable with mean  $\mu$  and variance  $\sigma^2$ . Also, let  $k$  be some positive integer. Show that  $P[|X - \mu| \leq k\sigma] \geq 1 - \frac{1}{k^2}$ . [13]

**Question 3 [20 marks]**

- 3.1 Let  $X$  be a random with a probability density function  $f(x)$  and a moment-generating function denoted by  $m_X(t)$ . Show that  $m_X(t)$  packages all moments about the origin in a single expression. That is,  $m_X(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \mu'_k$ . [5]
- 3.2 Let  $X$  be a random variable whose moment-generating function, denoted by  $m_X(t)$ , exists. Show that its second cumulant ( $k_2$ ) is related to its first and second moments by the following relationship  $k_2 = \mu_2 - \mu_1^2$ . [5]
- 3.3 (a) Show that the cumulant-generating function of an exponential random variable ( $X$ ), with a mean  $\frac{1}{\lambda}$ , is  $K_X(t) = \ln \lambda - \ln[\lambda - t]$ . [5]  
(b) Use the cumulant-generating function provided above to find the variance of  $X$ . [5]

**Question 4 [30 marks]**

- 4.1 The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution function

$$F(x) = \begin{cases} 1 - e^{-8x} & \text{for } x > 0 \\ 0 & \text{otherwise,} \end{cases}$$

Derive the characteristic function of  $X$  and use it to find the mean of  $X$ . [8]

- 4.2 Let  $Y$  be continuous random variable with a probability density function  $f(y) > 0$ . Also, let  $U = h(Y)$ . Then show that

$$f_U(u) = f_Y(h^{-1}(u)) \frac{dh^{-1}(u)}{du},$$

[7]

- 4.3 Suppose that  $X_1$  and  $X_2$  have a joint pdf given by  $f(x_1, x_2) = 2e^{-(x_1+x_2)}$  for  $0 < x_1 < x_2 < \infty$ . Let  $Y_1 = X_1$  and  $Y_2 = X_1 + X_2$ .
- (a) Find  $f(y_1, y_2)$ ; [10]  
(b) Find  $f(y_1)$ ; [2]  
(c) Find  $f(y_2)$ ; [2]  
(d) Are  $Y_1$  and  $Y_2$  independent? [1]

**END OF QUESTION PAPER**