



**NAMIBIA UNIVERSITY**  
OF SCIENCE AND TECHNOLOGY

**FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES**

**DEPARTMENT OF MATHEMATICS AND STATISTICS**

<b>QUALIFICATION:</b> Bachelor of Science in Applied Mathematics and Statistics	
<b>QUALIFICATION CODE:</b> 07BSOC; 07BAMS	<b>LEVEL:</b> 7
<b>COURSE CODE:</b> RAN701S	<b>COURSE NAME:</b> REAL ANALYSIS
<b>SESSION:</b> JUNE 2022	<b>PAPER:</b> THEORY
<b>DURATION:</b> 3 HOURS	<b>MARKS:</b> 100

**FIRST OPPORTUNITY EXAMINATION QUESTION PAPER**

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<b>MODERATOR:</b>	PROF FORTUNE' MASSAMBA

**INSTRUCTIONS**

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

**PERMISSIBLE MATERIALS**

1. Non-programmable calculator without a cover.

**THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)**

### QUESTION 1

Let  $(x_n)$  be a sequence of real numbers and  $x \in \mathbb{R}$ .

1.1. Define what does it mean to say the sequence  $(x_n)$  converges to  $x$ ? [2]

1.2. Use the definition in (1.1) to establish the sequence  $\left(\frac{\sin(\sqrt{n})}{\sqrt{n}}\right)$  converges to 0. [9]

### QUESTION 2

Find  $\lim_{n \rightarrow \infty} \left(\frac{n - \sqrt{n}}{\sqrt{n} + n}\right)$ . [5]

### QUESTION 3

3.1. Define what does it mean to say a sequence  $(x_n)$  in  $\mathbb{R}$  is a Cauchy sequence? [3]

3.2. Use the definition in (3.1.) to show that the sequence  $\left(\frac{n^2 + 3n}{n^2}\right)$  is a Cauchy sequence. [14]

### QUESTION 4

4.1 Determine the sum of  $\sum_{n=0}^{\infty} \frac{6}{(n+6)(n+7)}$  using partial fraction decomposition. [14]

4.2. Determine whether the series  $\sum_{n=1}^{\infty} (-1)^n \frac{3n+2}{\sqrt[3]{n^7+2n+1}}$  converges absolutely or conditionally. [10]

### QUESTION 5

Use the Epsilon- delta ( $\epsilon - \delta$ ) definition to show that  $\lim_{x \rightarrow 1} \frac{x-4}{x+2} = -1$ . [13]

### QUESTION 6

Show, using the definition of uniform continuity, the function  $f(x) = \frac{x}{x+1}$  is uniformly continuous on  $[0, 3]$ . [10]

### QUESTION 7

Apply the mean value theorem to prove that  $|\sin y - \sin x| \leq |y - x|$  for all  $x, y \in \mathbb{R}$ . [7]

**QUESTION 8**

8.1. Find the fifth degree Taylor polynomial  $p_5(x)$  of  $f(x) = e^x$  centered at 0. [7]

8.2. Determine a bound for the error when  $e^{0.5}$  approximated by  $p_5(x)$ . [6]

**END OF FIRST OPPORTUNITY EXAMINATION QUESTION PAPER**