



<b>QUALIFICATIONS: BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS AND BACHELOR of SCIENCE</b>	
<b>QUALIFICATION CODE: 07BSAM ,07BSOC</b>	<b>LEVEL: 6</b>
<b>COURSE: ORDINARY DIFFERENTIAL EQUATIONS</b>	<b>COURSE CODE: ODE602S</b>
<b>DATE: NOVEMBER 2023</b>	<b>SESSION: 1</b>
<b>DURATION: 3 HOURS</b>	<b>MARKS: 80</b>

**FIRST OPPORTUNITY: EXAMINATION QUESTION PAPER**

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**MODERATOR:** *Prof Sunday A. Reju*

**INSTRUCTIONS**

1. Answer any four questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left-side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Show all your working /calculation steps.

**PERMISSIBLE MATERIALS**

1. Non-Programmable Calculator

**ATTACHMENTS**

1. None

**This paper consists of 3 pages including this front page**

1. Solve the following initial value problems:

(a)  $y'(x) + \frac{4}{x}y(x) = 6x - 5, \quad y(1) = 1, \quad \text{for } x > 0$  (5)

(b)  $y'(x) + y(x) \tan x = e^{2x} \cos x, \quad y(0) = 2$  (5)

(c) Cobalt-60, a radioactive element employed in medical radiology, possesses a half-life of 5.3 years. Let's consider an initial cobalt-60 sample weighing 100 grams.

i. Calculate the decay constant and derive an equation representing the quantity of the sample that will remain  $t$  years from now. (5)

ii. What is the time required for 85% of the sample to undergo decay? (5)

2. (a) Find the values of  $\alpha$  such that  $y(x) = e^{\alpha x}$  is a solution of

$$y''(x) - y'(x) - 6y(x) = 0.$$

Determine if the solutions are linearly independent or not. Hence or otherwise, write the general solution. (6)

(b) Given that

$$ay''(x) + by'(x) + cy(x) = 0$$

i. Write down the auxiliary equation. (2)

ii. If the roots of the auxiliary equation are complex and denoted by  $m_1 = \alpha + \beta i$  and  $m_2 = \alpha - \beta i$ , show that the general solution is

$$y(x) = e^{\alpha x} (A \cos \beta x + B \sin \beta x)$$
 (6)

(c) Find the particular solution of the following differential equations, using undetermined coefficients

$$y''(x) - 6y'(x) + 8y(x) = 3 \cos x$$
 (6)

3. (a) Find the general solution of

$$0.5y^{iv}(x) + y''(x) + 0.5y = 0$$
 (6)

(b) Find the general solution of

$$2y'''(x) + 6y''(x) - 8y = 0$$
 (6)

(c) Find the general solution of

$$18x^2y''(x) + 30xy'(x) + 10y(x) = 0, \quad x > 0$$

4. (a) Use Laplace Transform to solve the differential equation:

$$y''(t) - 4y(t) = 24 \cos 2t, \quad y(0) = 3, \quad y'(0) = 4 \quad (10)$$

- (b) Solve by using Laplace Transform the following simultaneous differential equations:

$$x'(t) = x(t) - 2y(t), \quad \text{and} \quad y'(t) = 5x(t) - y(t), \quad x(0) = -1, \quad y(0) = 2 \quad (10)$$

5. (a) Use Laplace transform to find the exact value of

$$\int_0^{\infty} \frac{\cos 6t - \cos 4t}{t} dt. \quad (5)$$

- (b) Find the first five terms in the series solution of

$$y'(x) + y(x) + x^2 y(x) = \sin x, \quad \text{with} \quad y(0) = a. \quad (5)$$

- (c) If  $f(t) = e^{3t}$  and  $g(t) = e^{7t}$

i. Find the convolution of  $f(t) \otimes g(t)$  (5)

ii. Find  $\mathcal{L}\{f(t) \otimes g(t)\}$  (5)

**End of Exam!**