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| QUALIFICATION : **BACHELOR OF SCIENCE IN APPLIED MATHEMATICS AND STATISTICS** | |
| QUALIFICATION CODE: **07BSAM** | LEVEL: **7** |
| COURSE: **NUMERICAL METHODS 2** | COURSE CODE: **NUM702S** |
| DATE: **JANUARY 2024** | SESSION: **1** |
| DURATION: **3 HOURS** | MARKS: **90** |

**SECOND OPPORTUNITY / SUPPLEMENTARY: EXAMINATION QUESTION PAPER**

**EXAMINER:** *Dr S.N. NEOSSI-NGUETCHUE*

**MODERATOR:** *Prof S.S. MOTSA*

**INSTRUCTIONS:**

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Show clearly all the steps used in the calculations. All numerical results must be given using 5 decimals where necessary unless mentioned otherwise.
6. Mark all answers clearly with their respective question numbers.

**PERMISSIBLE MATERIALS:**

1. Non-Programmable Calculator

**ATTACHEMENTS**

None

**This paper consists of 3 pages including this front page**

**Problem 1** [25 Marks]

**1-1.** Show that the formula for the best line to fit data $(k, y_k)$ at integers $k$ for $1 \leq k \leq n$ is $y = ax + b$, where [15]

$$a = \frac{6}{n(n^2-1)}\left[2\sum_{k=1}^{n} ky_k - (n+1)\sum_{k=1}^{n} y_k\right]$$
$$b = \frac{4}{n(n-1)}\left[(2n+1)\sum_{k=1}^{n} y_k - 3\sum_{k=1}^{n} ky_k\right]$$

**1-2.** Establish the Padé approximation $e^x \approx R_{2,2}(x) = \dfrac{12+6x+x^2}{12-6x+x^2}$ and express $R_{2,2}$ in continued fraction form. [10]

**Problem 2** [20 Marks]

For any non negative interger $n$ we define Chebyshev polynomial of the first kind as

$$T_n(x) = \cos(n\theta), \quad \text{where } \theta = \arccos(x), \text{ for } x \in [-1, 1].$$

**2-1.** Show that the Chebyshev polynomial $T_n$ is a solution of the differential equation: [8]

$$(1-x^2)\frac{d^2f}{dx^2} - x\frac{df}{dx} + n^2 f = 0.$$

**2-2.** Compute the expressions of the first five Chebyshev polynomials of the first kind $T_0, T_1, T_2, T_3$ and $T_4$. [4]

**2-3.** Given the trucated power series $f(x) = 1 - x - x^3$.
(i) Economise the power series $f(x)$. [3]
(ii) Find the Chebyshev series for $f(x)$. [5]

**Problem 3** [13 Marks]

**3-1.** Given the integral

$$\int_0^3 \frac{\sin(2x)}{1+x^5}dx = 0.6717578646\cdots$$

**3-1-1.** Compute $T(J) = R(J, 0)$ for $J = 0, 1, 2, 3$ using the sequential trapezoidal rule. [10]

**3-2.** State the three-point Gaussian Rule for a continuous function $f$ on the interval $[-1, 1]$. [3]

**Problem 4** [32 Marks]

**4-1.** Assume a $3 \times 3$ matrix $A$ is known to have three different real eigenvalues $\lambda_1, \lambda_2$ and $\lambda_3$. Assume we know that $\lambda_1$ is near $-2$, $\lambda_2$ is near $-5$ and $\lambda_3$ is near $-1$.

**4-1-1.** Explain how the power method can be used to find the values of $\lambda_1$, $\lambda_2$ and $\lambda_3$ respectively. [2×3=6]

**4-1-2.** Discuss how shifting can be used in **4-1-1.** to accelerate the convergence of the power method. [2]

**4-2.** The matrix $A$ and its inverse are $A^{-1}$ are given below

$$A = \begin{bmatrix} 1/2 & -1 \\ -1 & 1 \end{bmatrix}, \qquad A^{-1} = \begin{bmatrix} -2 & -2 \\ -2 & -1 \end{bmatrix}.$$

- Use the power method to find the eigenvalue of the matrix $A$ with the smallest absolute value. Start with the vector $\mathbf{x}^{(0)} = (1, 0)^T$ and perform two iterations. [6]

**4-3.** Use Jacobi's method to find the eigenpairs of the matrix

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

[18]

God bless you !!!