QUALIFICATION : BACHELOR OF SCIENCE IN APPLIED MATHEMATICS AND STATISTICS	
QUALIFICATION CODE: **07BSAM; 07BSOC**	LEVEL: **7**
COURSE: **COMPLEX ANALYSIS**	COURSE CODE: **CAN702S**
DATE: **JANUARY 2024**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

SECOND OPPORTUNITY/SUPPLEMENTARY EXAMINATION: MEMORANDUM

EXAMINER:	DR. NEGA CHERE
MODERATOR:	PROF. FORTUNÉ MASSAMBA

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly with black or blue ink pen.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

ATTACHMENTS:

NONE

This paper consists of 2 pages including this front page.

1. (a) Find the real and imiginary part of $\frac{z+2}{z-2}$. (6)

   (b) Compute $\lim_{z \to 1+i} \frac{z^2 - 2iz + 1 - i}{z - 2 + i}$ if it exists. (4)

2. Let $z_1 = -1 - i, z_2 = 1 - i\sqrt{3}$. Then find the polar representation of $\frac{z_2}{z_1}$. (10)

3. Find the image of the disk $| z + 1 | < 2$ under the transformation
   $w = (1 + 2i)z + 2 - i$. (5)

4. Show that $\lim_{z \to 0} \frac{z^2}{|z|^2}$ does not exist, $(z = x + iy)$. (8)

5. Let $f(z) = f(x + iy) = 3x^2 - 2xy + x - 3y^2 + 2y + i(-x^2 - 6xy - 2x + y^2 + y)$. Determine
   if f is analytic in $\mathbb{C}$ or it is not analytic in $\mathbb{C}$. (12)

6. Show that $u(x, y) = y^3 - 4xy - 3x^2y$ is harmonic and find its harmonic conjugate $v(x, y)$
   for which $f(z) = u(x, y) + iv(x, y)$ is analytic. (15)

7. Evaluate $\int_C (xy - iy^2)dz$ where $(z = x + iy)$ and C is the counter joining 0 to $1+i, 1+i$
   to i and i to $-1 - i$. (20)

8. Evaluate the follwoing integrals.

   (a) $\int_C (\frac{z^2}{4 - z^2}) dz$ where C is the circle $| z + 1 | = 2$ oriented positively. (12)

   (b) $\int_C \frac{dz}{z^3(z+i)}$ where C is the circle $| z | = \frac{1}{2}$ oriented positively. (8)

END OF SECOND OPPORTUNITY/SUPPLEMENTARY EXAMINATION
QUESTION PAPER