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QUALIFICATION : **BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS**	
QUALIFICATION CODE: **07BSAM**	LEVEL: **7**
COURSE: **MATHEMATICAL MODELLING 2**	COURSE CODE: **MMO702S**
DATE: **NOVEMBER 2023**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **225** (To be converted to 100%)

FIRST OPPORTUNITY: QUESTION PAPER

EXAMINER: ***Prof Sunday A. Reju***

MODERATOR: ***Prof Oluwole D. Makinde***

INSTRUCTIONS

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.
6. Use of COMMA is NOT ALLOWED for a DECIMAL POINT.

PERMISSIBLE MATERIALS

1. Non-Programmable Calculator

ATTACHMENTS

NONE

This paper consists of 3 pages including this front page.

QUESTION 1 [97 MARKS]

(a) Discuss the role of Simulation Modelling as an extension of conventional Scientific Method with an appropriate diagram to reflect Mathematical modelling as a simulation technique. (7 Marks)

(b) Define the Linear Congruential Generator (LCG), and using a seed 3, multiplier 17, increment 2 and modulus 80, obtain the sequence of fifteen pseudo-random numbers using the LCG. (33 Marks)

Define cycling property and state if it occurs in the generated sequence, indicating when it occurs and the first two cycled pseudo-random numbers. (3 Marks)

(c) A customised LAN Email-to-Fax application delivers a block of textual data every 10 microseconds (µs). A conversion application checks each data block for conversion errors and corrects the errors, if necessary, before spontaneous conversion. It takes 1 µs to determine whether the block has any errors. If the block has one error, it takes 5µs to correct it and if it has more than 1 error it takes 20µs to correct the error. Blocks are queued when the converter falls behind. Assume that the converter is initially empty and that the number of errors in the first 15 blocks are: 1, 0, 3, 1, 0, 4, 0, 1, 0, 3, 1, 2, 0, 2, 1.

Construct a data conversion simulation table for the queueing model, showing arrival times, number of errors, waiting, conversion (service) and departure times. (45 Marks)

(d) From your simulation table in (c), determine the following performance measures (correct to 2 decimal places for non-integer numbers): (12 Marks, 2 Marks each)

(i) Average number of data blocks in the system.
(ii) Average block waiting time.
(iii) Maximum data conversion time.
(iv)Decoder busy duration.
(v) Decoder utilisation time.
(vi) Decoder idle time.

QUESTION 2 [32 MARKS]

(a) AGRIMAN Windhoek produces two farming implements: hoes and shovels and realises a net unit profit of N$115.50 per hoe and N$120.65 per shovel. Assume that the firm has up to 130 square metres of iron sheet and 120 metres of wood length to devote to a small farming project plus a signed contract of supplying 10 hoes and 15 shovels to a Rehoboth farm during the period of the project. In addition, assume that it requires 2.5 square metres of iron and 1.65 metre of wood to fabricate a hoe and 1.2 square metres of iron and 1.85 metre of wood to produce a shovel. Formulate and solve the model for maximising the firm's profits during the project, stating also how many of each of the implements will the firm produce for the project apart from the contract. (15 Marks)

(b) (i) Define post-optimality analysis for linear optimisation problems. (5 Marks)

(ii) Discuss the analysis for change in the firm's profits on hoes, showing all expressions to support your conclusion. (12 Marks)

QUESTION 3 [54 MARKS]

(a) Discuss the Method of Substitution for solving nonlinear optimisation problems.

(2 Marks)

Hence use the method to solve the following problems:

(i) Minimise $f(x_1, x_2) = 4x_1^2 + 5x_2^2$ (13 Marks)

subject to $2x_1 + 3x_2 = 6$

(ii) Maximise $z(x_1, x_2) = 4x_1 - 0.1x_1^2 + 5x_2 - 0.2x_2^2$ (14 Marks)

subject to $x_1 + 2x_2 = 40$

(b)

(i) Solve the problem in (a)(i) above using the Lagrange Multiplier's method.

(10.5 Marks)

(ii) State the theorem for necessary and sufficient conditions for optimality for nonlinear optimisation problems and specifically the Karush-Kuhn-Tucker (KKT) conditions within the theorem. (14.5 Marks)

QUESTION 4 [42 MARKS]

Suppose a large lake that was formed by building a dam over a river holds initially 100 million gallons of water. Because a nearby agricultural field was sprayed with a pesticide, the water has become polluted. The concentration of the pesticide has been measured and is equal to 35ppm (parts per million), or 35×10^{-6}.

The river continues to flow into the lake at a rate of 300 gal/min. The river is only slightly polluted with a pesticide and has a concentration of 5 ppm. The flow of water over the dam can be controlled and is set at 400 gal/min. Assume that no additional spraying causes the lake to become even more polluted.

Formulate the pollution model and hence determine how long will it be before the lake water reaches an acceptable level of pesticide concentration equal to 15 ppm.

END OF EXAMINATION TOTAL MARKS:225 (CONVERT TO 100%)