QUALIFICATIONS :	BACHELOR of SCIENCE HONOURS IN APPLIED MATHEMATICS
	BACHELOR of SCIENCE HONOURS IN APPLIED STATISTICS

QUALIFICATION CODES: 08BSMH, 08BSSH	LEVEL: 8
COURSE: APPLIED OPERATIONS RESEARCH	COURSE CODE: AOR802S
DATE: NOVEMBER 2024	SESSION: 1
DURATION: 3 HOURS	MARKS: 140 (To be converted to 100%)

FIRST OPPORTUNITY: EXAMINATION QUESTION PAPER

EXAMINER:	*Prof Sunday A. Reju*
MODERATOR:	*Prof Oluwole D. Makinde*

INSTRUCTIONS

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.
6. Use of COMMA is NOT ALLOWED for a DECIMAL POINT.

PERMISSIBLE MATERIALS

1. Non-Programmable Calculator

ATTACHMENTS

NONE

This paper consists of 4 pages including this front page.

QUESTION 1 [25 MARKS]

(a) Consider a competition between two companies, Coca-Cola and Pepsi, and assume the former is thinking of cutting the price of its iconic soda. If it does so, Pepsi may have no choice but to follow suit for its cola to retain its market share. This may result in a significant drop in profits for both companies. Let's assume that the incremental profits that accrue to Coca-Cola and Pepsi are as follows: If both keep prices high, profits for each company increase by $500 million (because of normal growth in demand). If one drops prices (i.e. defects) but the other does not (i.e. cooperates), profits increase by $750 million for the former because of greater market share and are unchanged for the latter. If both companies reduce prices, the increase in soft drink consumption offsets the lower price, and profits for each company increase by $250 million.   (13 marks)

    (i)   Construct the payoff matrix for the game model, taking Coca-Cola as the row player and each company's payoff matrix.

    (ii) What should each company do?

(b) Suspects A and B have been apprehended for a crime and are in cells in Outapi police station, with no means of communicating with each other. The prosecutor has separately told them the following: (12 marks)

    "If you confess and agree to testify against the other suspect, who does not confess, the charges against you will be dropped and you will go scot-free. If you do not confess but the other suspect does, you will be convicted, and the prosecution will seek the maximum sentence of three years. If both of you confess, you will both be sentenced to two years in prison. If neither of you confesses, you will both be charged with misdemeanours and will be sentenced to one year in prison".

(i) Selecting Suspect A as the row player in a 2-person game, construct the game payoff matrix and each suspect's payoff matrix to determine what the two suspects should do and discuss fully why.

(ii) Discuss the implication of the dominant strategy for each prisoner.

QUESTION 2 [30 MARKS]

(a) Discuss the Reduction by Dominance procedure and hence simplify by using reduction by dominance the game defined by the following payoff matrix, showing progressively the reduced pay-off matrix:

$$
A \quad
\begin{array}{c}
 \\
 \\
 \\
 \\
\end{array}
\begin{array}{c}
\quad\quad B \\
\begin{array}{ccc}
a & b & c
\end{array} \\
\begin{array}{c}
A \\ B \\ C \\ D
\end{array}
\begin{bmatrix}
1 & -1 & -5 \\
4 & -4 & 2 \\
3 & -3 & -10 \\
5 & -5 & -4
\end{bmatrix}
\end{array}
$$

(8 Marks)

(b) Distinguish between pure and mixed strategies.   (5 Marks)

(c)   Eugene has a 250-gallon capacity home heating oil tank, presently empty, meant to store oil against the next winter. Consider the following winter heating oil quantity needed and the oil prices during probable four levels of winter severity:

WINTER SEVERITY	OIL STORAGE NEEDED	OIL PRICES PER GALLON
Mild Winter (MW)	110 Gallons	N$1.00
Average Winter (AW)	180 Gallons	N$1.85
Severe Winter (SW)	230 Gallons	N$2.00
Prolonged Winter (PW)	250 Gallons	N$3.00

Formulate a game model and employ the Minimax criterion technique to determine the gallons of oil Eugene should stockpile at the current price of N$1 per gallon to avoid wasted unused oil and to maximise his saving.   (17 Marks)

QUESTION 3 [25 MARKS]

(a)   A company licensed to mine in possession of a natural resource field has 0.25 chance for diamond discovery. However, the company has the options to either mine the resource or to sell it the field to another mining company wishing to buy the land for $90,000,000. The cost of mining by the licensee is $100,000,000 with a revenue yield of $800,000,000 if diamond is found. Formulate a game of strategy model to perform a mathematical decision analysis of the problem and determine the decision of the licensee, stating appropriate assumptions for your method.   (8 Marks)

(b) State the Maximum Likelihood Criterion and confirm the above decision obtained in (a) with the criterion.   (4 Marks)

(c) Assuming the company feels that the true chances of discovering diamond are likely to be between 15% and 35%, define sensitivity analysis and the decision crossover point, and hence show that the decision is sensitive to these prior probabilities, providing an appropriate sketch to substantiate your decision analysis and conclusion.   (13 Marks)

QUESTION 4 [60 MARKS]

(a)   Consider a winning bid of $5.4 million to construct a new plant for a major manufacturer and the manufacturer needs the plant to go into operation within 40 weeks. Below is the list of the various project activities. The third column provides important additional information for coordinating the scheduling of the project crews.

Activity	Activity Description	Immediate Predecessors	Estimated Duration
A	Excavate	—	2 weeks
B	Lay the foundation	A	4 weeks
C	Put up the rough wall	B	10 weeks
D	Put up the roof	C	6 weeks
E	Install the exterior plumbing	C	4 weeks
F	Install the interior plumbing	E	5 weeks
G	Put up the exterior siding	D	7 weeks
H	Do the exterior painting	E, G	9 weeks
I	Do the electrical work	C	7 weeks
J	Put up the wallboard	F, I	8 weeks
K	Install the flooring	J	4 weeks
L	Do the interior painting	J	5 weeks
M	Install the exterior fixtures	H	2 weeks
N	Install the interior fixtures	K, L	6 weeks

(i) Define Critical Path Method (CPM) and Project Evaluation and Review Technique (PERT).

(3 Marks)

(ii) Sketch the project network diagram for the above project. (16 Marks)

(iii) Distinguish between crashing a project and a project activity. Hence obtain the crash costs per week saved for each activity from the following investigative time-cost trade-off data. (37 Marks)

Activity	Time Normal	Time Crash	Cost Normal	Cost Crash	Activity	Time Normal	Time Crash	Cost Normal	Cost Crash
A	2 weeks	1 week	$180,000	$280,000	I	7 weeks	5 weeks	$210,000	$270,000
B	4 weeks	2 weeks	$320,000	$420,000	J	8 weeks	6 weeks	$430,000	$490,000
C	10 weeks	7 weeks	$620,000	$860,000	K	4 weeks	3 weeks	$160,000	$200,000
D	6 weeks	4 weeks	$260,000	$340,000	L	5 weeks	3 weeks	$250,000	$350,000
E	4 weeks	3 weeks	$410,000	$570,000	M	2 weeks	1 week	$100,000	$200,000
F	5 weeks	3 weeks	$180,000	$260,000	N	6 weeks	3 weeks	$330,000	$510,000
G	7 weeks	4 weeks	$900,000	$1,020,000					

(iv) Discuss your observations. (4 Marks)

END OF EXAMINATION   TOTAL MARKS = 140