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QUALIFICATION : **BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS**	
QUALIFICATION CODE: **07BSAM**	LEVEL: **5**
COURSE: **LINEAR ALGEBRA 1**	COURSE CODE: **LIA502S**
DATE: **JANUARY 2024**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

SECOND OPPORTUNITY / SUPPLEMENTARY: EXAMINATION QUESTION PAPER

EXAMINER: MR GABRIEL S MBOKOMA, DR NEGA CHERE

MODERATOR: DR DAVID IIYAMBO

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

This paper consists of 3 pages including this front page.

Question 1

Consider the vectors $\mathbf{p} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{q} = \mathbf{i} - 3\mathbf{j} + 12\mathbf{k}$

a) Find the unit vector in the direction of **p**. [4]

b) Find the angle (*in degrees*) between **p** and **q**. Give you answer correct to 1 d.p. [8]

Question 2

2.1 Write down a 4×4 matrix whose ij^{th} entry is given by $a_{ij} = \frac{1}{ij+1}$, and comment on your matrix. [6]

2.2 Let A be a square matrix. State what is meant by each of the following statements.

a) A is symmetric [2]

b) A is orthogonal [2]

c) A is skew-symmetric [2]

2.3 Consider the following matrices.

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{pmatrix}, \quad \text{and } D = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{pmatrix}.$$

a) Given that $C = AB$, determine the element c_{32}. [5]

b) Find $(3A)^T$. [5]

c) Is DB defined? If yes, then find it, and hence calculate $tr(DB)$. [6]

2.4 Suppose A is a square matrix. Check if the matrix $B = 3(A - A^T)$ is skew-symmetric? [5]

Question 3

Consider

3.1 Let

$$B = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}.$$

Show that the $\det(B) = 1$. [4]

3.2 Consider

$$A = \begin{bmatrix} 2 & 0 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 4 \end{bmatrix}.$$

Use the adjoint matrix of A to find A^{-1}. [10]

Question 4

Use the *Crammer's rule* to solve the following system of linear equations, if it exists.

$$\begin{aligned} 2x - y + 3z &= 2 \\ 3x + y - 2z &= 0 \\ 2x - 2y + z &= 8 \end{aligned}$$

[12]

Question 5

a) Prove that in a vector space, the negative of each vector is unique. [9]

b) Determine whether the following set is a subspace of $\mathbb{R}^3$.

$$S = \{(x,\, y,\, z) \in \mathbb{R}^3 \mid x + y + z = 0\}$$

[12]

c) Prove that if $\mathbf{x}$ and $\mathbf{y}$ are orthogonal vectors in $\mathbb{R}^n$, then show that

$$\|\mathbf{x} + \mathbf{y}\|^2 = \|\mathbf{x}\|^2 + \|\mathbf{y}\|^2$$

[8]
