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QUALIFICATION : **BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS**	
QUALIFICATION CODE: **07BSAM**	LEVEL: **5**
COURSE: **LINEAR ALGEBRA 1**	COURSE CODE: **LIA502S**
DATE: **JANUARY 2025**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

SECOND OPPORTUNITY / SUPPLEMENTARY: EXAMINATION QUESTION PAPER

EXAMINER:	MR GABRIEL S MBOKOMA, MR ILENIKEMANYA NDADI
MODERATOR:	DR DAVID IIYAMBO

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

This paper consists of 2 pages including this front page.

Question 1

Consider the vectors $\mathbf{v} = 4\mathbf{i} - 8\mathbf{k}$, $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

a) Find a vector of magnitude $\sqrt{5}$ in the direction of $\mathbf{v}$. [6]

b) Find the angle θ (in radians) that is between $\mathbf{a}$ and $\mathbf{b}$. [6]

c) Find a unit vector that is perpendicular to both vectors $\mathbf{a}$ and $\mathbf{b}$. [8]

Question 2

Let A be a square matrix and let

$$S = \frac{1}{2}(A + A^T) \quad \text{and} \quad P = \frac{1}{2}(A - A^T).$$

a) Find S+P. [4]

b) Show that S is symmetric and P is skew-symmetric. [6]

c) Show that if A is symmetric, then $S = A$ and $P = 0$. [4]

Question 3

Consider the matrix $B = \begin{pmatrix} 1 & 2 & 1 \\ 3 & -2 & -4 \\ 2 & 3 & -1 \end{pmatrix}$.

a) Use the *Cofactor expansion method*, expanding along the first row, to evaluate the determinant of B. [9]

b) Is B invertible? If it is, find B^{-1}. [14]

c) Find $\det\left(((2B)^{-1})^T\right)$. [6]

Question 4

Show that $\mathbf{u_1} = (1, 1, 1), \mathbf{u_2} = (1, 2, 3)$ and $\mathbf{u_3} = (1, 5, 8)$ span $\mathbb{R}^3$ (use Gaussian). [15]

Question 5

a) Prove that in a vector space, the negative of a vector is unique. [9]

b) Determine whether the following set is a subspace of $\mathbb{R}^n$.

$$S = \{(a,\ b,\ c) \in \mathbb{R}^n \mid a + b + c = 0\}$$

[13]
