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QUALIFICATION : **BACHELOR OF SCIENCE IN APPLIED MATHEMATICS AND STATISTICS**	
QUALIFICATION CODE: **07BSAM; 07BSOC**	LEVEL: **6**
COURSE: **LINEAR ALGEBRA 2**	COURSE CODE: **LIA601S**
DATE: **JANUARY 2024**	SESSION: **1**
DURATION: **3 HOURS**	MARKS: **100**

SECOND OPPORTUNITY/SUPPLEMENTARY EXAMINATION: QUESTION PAPER

EXAMINER: DR. NEGA CHERE

MODERATOR: DR. DAVID IIYAMBO

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly with black or blue ink pen.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

ATTACHMENTS:

NONE

This paper consists of 3 pages including this front page.

Part I: True or false questions.

For each of the following questions, state whether it is true or false. Justify your answer.

1. The map $T : \mathbb{R}^3 \to \mathbb{R}^2$, defined by T(x,y,z) = (x + y +2, y + z) is not a linear transformation. (3)

2. If A and B are similar matrices then there exists an invertible matrix P such that AP = BP. (3)

3. For an $n \times n$ matrix A, the geometric multiplicity of each eigenvalue of A is less than or equal to the algebraic multiplicity. (3)

4. The index and signature of the quadratic form $q(x, y, z) = 3x^2 - 4xy + 6y^2 + 4yz - 7z^2$ are respectively 3 and 2. (3)

5. If q is a quadratic form on a vector space V, then $q(-\alpha) = -q(\alpha)$. (3)

Part II: Work out problems.

1. Let V and W be vector spaces over a field K and let T: V $\to$ W be a mapping. State what it means to say T is linear transformation. (3)

2. Let T be the mapping $T : P_3 \to P_2$ defined by $T(a_0 + a_1x + a_2x^2 + a_3x^3) = 2a_1 - a_2x^3$. Then
   (a) show that T is linear. (12)
   (b) find a basis for the kernel of T. (7)

3. Let V be the vector space of functions with basis $S = \{sin\, 2t, cos\, 2t, e^{-3t}\}$ and let D: $V \to V$ be the differential operator defined by $D\, f(t) = \frac{d}{dt}f(t)$. Find the matrix representing D in the basis S. (8)

4. Let A and B be $n \times n$ similar matrices.Then prove that A and B have the same determinant. (6)

5. Consider the bases $\mathcal{B} = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ and $\mathcal{C} = \{(1, 0, 1), (0, 1, 1), (1, 1, 0)\}$ of $\mathbb{R}^3$.
   (a) Find the change of basis matrix $P_{\mathcal{C} \leftarrow \mathcal{B}}$ from $\mathcal{B}$ to $\mathcal{C}$. (10)
   (b) Use the result in (a) and to compute $[v]_{\mathcal{C}}$ where $v = (1, 3, 5)$. (5)

6. (a) Show that $\lambda = 4$ is an eigenvalue of the matrix A= $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 2 & 1 \\ 3 & 0 & 1 \end{pmatrix}$ and find an eigenvector corresponding to this eigenvalue. (17)
   (b) Show that v = $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ is an eigenvector for the matrix A = $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & -2 \\ 1 & 0 & 1 \end{pmatrix}$ and find the corresponding eigenvalue of A. (6)

7. (a) Consider the bilinear form f on $\mathbb{R}^2$ defined by $f((x_1, y_1), (x_2, y_2)) = 2x_1x_2 - 3x_1y_2 + 4y_1y_2$. Find the matrix A of f relative to the basis $\mathrm{B} = \{(1, 1), (-2, 1)\}$. (6)

   (b) Show that $q(x, y) = x^2 + 2xy + y^2$ is a quadratic form on $\mathbb{R}^2$. (5)

END OF SECOND OPPORTUNITY/SUPPLEMENTARY EXAMINATION QUESTION PAPER