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QUALIFICATION : **BACHELOR of SCIENCE IN APPLIED MATHEMATICS AND STATISTICS**	
QUALIFICATION CODE: **07BSAM; 07BSOC**	LEVEL: **6**
COURSE: **LINEAR ALGEBRA 2**	COURSE CODE: **LIA601S**
DATE: **JANUARY 2025**	SESSION: **2**
DURATION: **3 HOURS**	MARKS: **100**

SECOND OPPORTUNITY / SUPPLEMENTARY: EXAMINATION QUESTION PAPER

EXAMINER: DR. NEGA CHERE

MODERATOR: DR. DAVID IIYAMBO

INSTRUCTIONS:

1. Answer all questions on the separate answer sheet.
2. Please write neatly and legibly.
3. Do not use the left side margin of the exam paper. This must be allowed for the examiner.
4. No books, notes and other additional aids are allowed.
5. Mark all answers clearly with their respective question numbers.

PERMISSIBLE MATERIALS:

1. Non-Programmable Calculator

ATTACHMENTS:

NONE

This paper consists of 3 pages including this front page.

QUESTION 1 [21]

1.1. Let $T: M_{nn} \to \mathbb{R}$ be a mapping defined by $T(A) = tr(A)$. Determine whether T is linear or not where M_{nn} is the set of all $n \times n$ matrices. [12]

1.2. Let $f: \mathbb{R}^3 \to \mathbb{R}^2$ be a mapping defined by $f(x, y, z) = \left(x + z,\ y - z,\ \sqrt{z}\right)$. Determine whether f is linear or not. [9]

QUESTION 2 [9]

Find the coordinate vector $[p(x)]_{\mathcal{B}}$ of $p(x) = 2 - x + x^2$ with respect to the ordered basis $\mathcal{B} = \{1 + x,\ 1 + x^2,\ x + x^2\}$ of P_2. [9]

QUESTION 3 [23]

Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be mapping defined by $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + 2x_2 - x_3 \\ x_2 + x_3 \\ x_1 + x_2 - 2x_3 \end{pmatrix}$.

3.1. Find the standard matrix for T and use it to determine $T(x)$ where $x = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$. [8]

3.2. Find a basis and the dimension of the image of T. Use rank-nullity theorem to determine the nullity of T and use it to determine whether T is singular or nonsingular. [15]

QUESTION 4 [12]

4.1. Let $A = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}$. Show that A and B are not similar. [4]

4.2. If λ is an eigenvalue of an invertible matrix A with corresponding eigenvector x, then show that λ^n is an eigenvalue of A^n with corresponding eigenvector x. [8]

QUESTION 5 [7]

Consider the following two bases of $\mathbb{R}^3$: $S = \{e_1, e_2, e_3\} = \{(1,0,0), (0,1,0), (0,0,1)\}$ and $E = \{v_1, v_2, v_3\} = \{(1,1,0), (0,1,1), (1,2,2)\}$. Find the change of basis matrix from S to E, $P_{E \leftarrow S}$. [7]

QUESTION 6 [28]

Let $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.

6.1. Determine the eigenvalues and the corresponding eigenvectors of A. [25]

6.2. Is matrix A diagonalizable? If it is, find an invertible matrix P that diagonalizes A. [3]

END OF SECOND OPPORTUNITY / SUPPLEMENTARY: EXAMINATION QUESTION PAPER