NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

SCHOOL OF NATURAL AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: Bachelor of Science Honours in Applied Statistics	
QUALIFICATION CODE: O8BSHS	**LEVEL:** 8
COURSE CODE: ASS 801S	**COURSE NAME:** APPLIED SPATIAL STATISTICS
SESSION: JULY 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr D. NTIRAMPEBA
MODERATOR:	Prof G. O. ORWA

INSTRUCTIONS
1. Answer ALL the questions in the booklet provided. 2. Show clearly all the steps used in the calculations. 3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

ATTACHMENTS

1. Chi-square table

THIS QUESTION PAPER CONSISTS OF 4 PAGES (Excluding this front page & Chi-square table)

Question 1 [20 marks]

1.1 (a) Briefly explain the following terminologies as they are applied to Spatial Statistics.
(i) Feature [2]
(ii) Support [2]
(iii) Local spillovers [2]
(iv) Global spillovers [2]
(v) Areal data [3]

(b) State Tobler's first law of geography. Use this law to explain briefly what the influence of this law will be in Spatial Statistics. [3]

1.2 Let $X_1, \ldots, X_n$ be random variables in ℓ^2. The symmetric covariance matrix of the random vector $\mathbf{X} = (X_1, \ldots, X_n)^T$ is defined by
$\Sigma := Cov(\mathbf{X}) = E[(\mathbf{X} - E(\mathbf{X}))(\mathbf{X} - E(\mathbf{X}))^T]$. Note that $\Sigma_{i,j} = Cov(X_i, X_j)$
(a) Show that Σ is positive semi-definite. [5]
(b) Define what it means for Σ to be a non-degenerate covariance matrix? [1]

Question 2 [20 marks]

2.1 Consider a vector of areal unit data $Z = (Z_1, \ldots, Z_n)$ relating to n non-overlapping areal units. Additionally, consider a binary $n \times n$ neighbourhood matrix W, where $w_{kj} = 1$ if areas (k, j) share a common border and $w_{kj} = 0$ otherwise.
(a) Define mathematically a Global Moran's I statistic and show how to compute the Z-score associated to it. [4]

(b) Data were obtained for the $n = 624$ electoral wards in Greater London for 2009 on the observed numbers of admissions to hospital due to respiratory disease(y, response variable). Also collected were two covariates, the percentage of people defined to be poor (poor) in each area, and the average air pollution concentrations (pollution) in each area.
(i) An initial simple Poisson generalised linear model was fitted to the hospital admission counts (y), with both covariates and the (log) expected numbers of admissions as a known offset term. The residuals were then tested for the presence of spatial autocorrelation, and the results of a Morans I test are shown below.

Monte-Carlo simulation of Moran I:
Data: res
Weights: W.list
Number of simulations + 1: 1001

statistic=0.39417, observed rank=1001, p-value=0.000999
alternative hypothesis: greater
What does this test tell you about the presence or abscence of residual spatial autocorrelation? Justify [2]

(ii) A Poisson log-linear Conditional Autoregressive(CAR) model was then fitted to these data, where the linear predictor contained a set of random effects $\phi = (\phi_1, \dots, \phi_n)$ in addition to the covariates. The CAR model used has full conditional distributions given by, $\phi_i|\phi_{-i} \sim N\left(\frac{\rho \sum_{j=1}^{n} w_{ij}\phi_j}{\rho \sum_{j=1}^{n} w_{kj}+(1-\rho)}, \frac{\tau^2}{\rho \sum_{j=1}^{n} w_{ij}+(1-\rho)}\right)$, where in the usual notation ϕ_{-i} denotes all the spatial effects except the ith.

Output from fitting the model is given below.

	Median	2.5%	97.5%
Intercept	-0.8512	-1.2169	-0.4939
pollution	0.0074	0.0104	0.0258
poor	0.0267	0.0239	0.0259
tau	0.0798	0.067	0.0945
rho	0.8324	0.6825	0.9492

What does the estimated value of ρ tell you about the level of residual spatial autocorrelation after adjusting for the covariates? Justify your answer [2]

(iii) Calculate separate relative risks and 95% credible intervals for increases in each covariate by 1 unit and interpret the results. [6]

(iv) How could the CAR model given above be simplified to the intrinsic CAR model for strong spatial autocorrelation? Provide the mathematical expression of the new model after simplification. [2]

2.2 Briefly compare spatial Lag and Spatial error models. [4]

Question 3 [33 marks]

3.1 Distinguish between strict stationarity, second order stationarity, and intrinsic hypotheses of a regionalised variable. [6]

3.2 Suppose that using two points on a straight line the value at a third point is to be estimated. The points are $s_1 = 1$ and$s_2 = -2$. The point for which the estimation is to be done is $s_0 = 0$. Fig.1 shows the data configuration. Let the measurement values be $Z(s_1) = 2$ and $Z(s_2) = 4$. Suppose the variogram is linear, that $\gamma(h) = h$

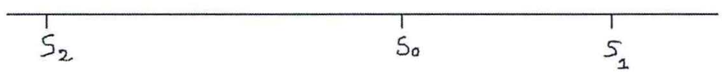


Figure 1: Data configuration 1

Use ordinary kriging to estimate $Z(s_0)$ and $\sigma^2_{Ok}(s_0)$ (the estimated variance). [6]

3.3 Let $\{Z(s) : s \in D\}$, $D \subset \Re$ be a geostatistical process with a wave covariance function given by

$$C_z(h) = \begin{cases} \tau^2 + \sigma^2 & \text{for } h = 0 \\ \sigma^2 \left[\frac{sin(\frac{h}{\phi})}{\frac{h}{\phi}}\right] & h > 0, \end{cases}$$

Derive:

(a) the expression of a wave semi-variogram function, [5]

(b) the correlation function for $\rho_Z(h)$. [3]

3.4 Let $\{Z(s) : s \in D\}$ be an **intrinsically stationary random function** with known variogram function $\gamma(h)$.

(a) Show that the predictor for ordinary kriging at unsampled location s_0 defined by

$$Z^*_{OK}(s_0) = \sum_{i=1}^{n} w_i Z(s_i)$$

is unbiased Estimator. [3]

(b) Show that the variance of the prediction error is given by

$\sigma_E^2 = Var(Z^*_{OK}(s_0) - Z(s_0)) = -\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \gamma(s_i - s_j) + 2\sum_{i=1}^{n} w_i \gamma(s_i - s_0)$

Hint:

$$-\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \frac{(Z(s_i) - Z(s_j))^2}{2} + 2\sum_{i=1}^{n} w_i \frac{(Z(s_i) - Z(s_0))^2}{2}$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Z(s_i)Z(s_j) - 2\sum_{i=1}^{n} w_i Z(s_i)Z(s_0) + (Z(s_0))^2$$

[10]

Question 4 [27 marks]

4.1 Let Z be a spatial point process in a spatial domain $D \in \Re^2$.

(a) Explain what is meant by saying that Z is:

(1) a Homogeneous Poisson Process(HPP). [3]

(2) a regular process [2]

(b) Describe briefly the difference between a **marked** and **unmarked** spatial point process [2]

4.2 Assume that Z is a Homogeneous Poisson Process(HPP) in a spatial domain $D \subset \Re^2$. Use the maximum likelihood estimation method to show the constant first order intensity function λ is given by $\lambda = \frac{Z(D)}{|D|} = \frac{n}{|D|}$. [10]

4.3 Consider a spatial point process $Z = \{Z(A) : A \subset D\}$, where D is the domain of interest.

(a) One hypothesis test of quantifying whether an observed spatial point pattern is completely spatially random is based on quadrat counts, write down the null and alternative hypotheses for this test, the test statistic, and the distribution of the test statistic under the null hypothesis. [4]

(b) Consider the following point process of $n = 101$ points, split into 9 quadrats containing 3 rows and 3 columns as shown if Figure.2. Use the method of quadrat counts to test whether the data are drawn from a complete spatial random process(show all steps involved in the hypothesis testing process). . [6]

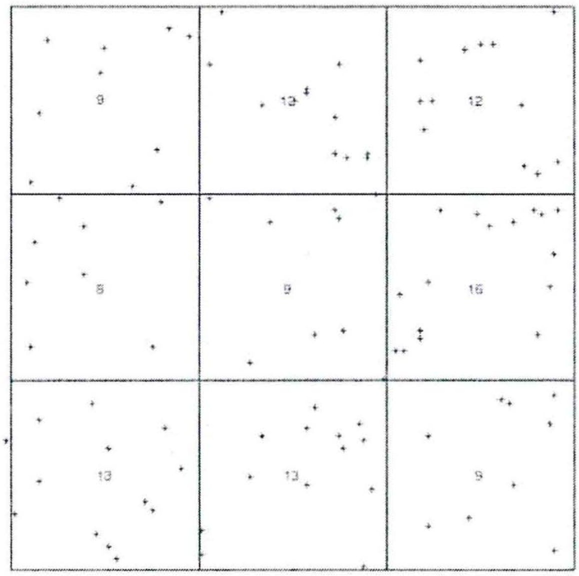
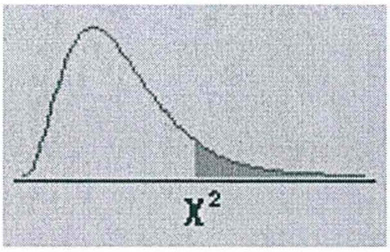


Figure 2: Distribution points partioned into 9 quadrats

END OF QUESTION PAPER

The Chi-Square Distribution



df\p	.995	.990	.975	.950	.900	.750	.500	.250	.100	.050	.025	.010	.005
1	0.00004	0.00016	0.00098	0.00393	0.01579	0.10153	0.45494	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
2	0.01003	0.02010	0.05064	0.10259	0.21072	0.57536	1.38629	2.77259	4.60517	5.99146	7.37776	9.21034	10.59663
3	0.07172	0.11483	0.21580	0.35185	0.58437	1.21253	2.36597	4.10834	6.25139	7.81473	9.34840	11.34487	12.83816
4	0.20699	0.29711	0.48442	0.71072	1.06362	1.92256	3.35669	5.38527	7.77944	9.48773	11.14329	13.27670	14.86026
5	0.41174	0.55430	0.83121	1.14548	1.61031	2.67460	4.35146	6.62568	9.23636	11.07050	12.83250	15.08627	16.74960
6	0.67573	0.87209	1.23734	1.63538	2.20413	3.45460	5.34812	7.84080	10.64464	12.59159	14.44938	16.81189	18.54758
7	0.98926	1.23904	1.68987	2.16735	2.83311	4.25485	6.34581	9.03715	12.01704	14.06714	16.01276	18.47531	20.27774
8	1.34441	1.64650	2.17973	2.73264	3.48954	5.07064	7.34412	10.21885	13.36157	15.50731	17.53455	20.09024	21.95495
9	1.73493	2.08790	2.70039	3.32511	4.16816	5.89883	8.34283	11.38875	14.68366	16.91898	19.02277	21.66599	23.58935
10	2.15586	2.55821	3.24697	3.94030	4.86518	6.73720	9.34182	12.54886	15.98718	18.30704	20.48318	23.20925	25.18818
11	2.60322	3.05348	3.81575	4.57481	5.57778	7.58414	10.34100	13.70069	17.27501	19.67514	21.92005	24.72497	26.75685
12	3.07382	3.57057	4.40379	5.22603	6.30380	8.43842	11.34032	14.84540	18.54935	21.02607	23.33666	26.21697	28.29952
13	3.56503	4.10692	5.00875	5.89186	7.04150	9.29907	12.33976	15.98391	19.81193	22.36203	24.73560	27.68825	29.81947
14	4.07467	4.66043	5.62873	6.57063	7.78953	10.16531	13.33927	17.11693	21.06414	23.68479	26.11895	29.14124	31.31935
15	4.60092	5.22935	6.26214	7.26094	8.54676	11.03654	14.33886	18.24509	22.30713	24.99579	27.48839	30.57791	32.80132
16	5.14221	5.81221	6.90766	7.96165	9.31224	11.91222	15.33850	19.36886	23.54183	26.29623	28.84535	31.99993	34.26719
17	5.69722	6.40776	7.56419	8.67176	10.08519	12.79193	16.33818	20.48868	24.76904	27.58711	30.19101	33.40866	35.71847
18	6.26480	7.01491	8.23075	9.39046	10.86494	13.67529	17.33790	21.60489	25.98942	28.86930	31.52638	34.80531	37.15645
19	6.84397	7.63273	8.90652	10.11701	11.65091	14.56200	18.33765	22.71781	27.20357	30.14353	32.85233	36.19087	38.58226
20	7.43384	8.26040	9.59078	10.85081	12.44261	15.45177	19.33743	23.82769	28.41198	31.41043	34.16961	37.56623	39.99685
21	8.03365	8.89720	10.28290	11.59131	13.23960	16.34438	20.33723	24.93478	29.61509	32.67057	35.47888	38.93217	41.40106
22	8.64272	9.54249	10.98232	12.33801	14.04149	17.23962	21.33704	26.03927	30.81328	33.92444	36.78071	40.28936	42.79565
23	9.26042	10.19572	11.68855	13.09051	14.84796	18.13730	22.33688	27.14134	32.00690	35.17246	38.07563	41.63840	44.18128
24	9.88623	10.85636	12.40115	13.84843	15.65868	19.03725	23.33673	28.24115	33.19624	36.41503	39.36408	42.97982	45.55851
25	10.51965	11.52398	13.11972	14.61141	16.47341	19.93934	24.33659	29.33885	34.38159	37.65248	40.64647	44.31410	46.92789
26	11.16024	12.19815	13.84390	15.37916	17.29188	20.84343	25.33646	30.43457	35.56317	38.88514	41.92317	45.64168	48.28988
27	11.80759	12.87850	14.57338	16.15140	18.11390	21.74940	26.33634	31.52841	36.74122	40.11327	43.19451	46.96294	49.64492
28	12.46134	13.56471	15.30786	16.92788	18.93924	22.65716	27.33623	32.62049	37.91592	41.33714	44.46079	48.27824	50.99338
29	13.12115	14.25645	16.04707	17.70837	19.76774	23.56659	28.33613	33.71091	39.08747	42.55697	45.72229	49.58788	52.33562
30	13.78672	14.95346	16.79077	18.49266	20.59923	24.47761	29.33603	34.79974	40.25602	43.77297	46.97924	50.89218	53.67196